# Glauber approximation and the problem of diffraction minima

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The influence of the spin terms and of the momentum-transfer dependence of the nucleon-nucleon amplitude phase on the behavior of the differential cross section for elastic nucleon-nucleus collisions in the region of the diffraction minimum is considered.

We consider a nucleus with zero spin, confine ourselves to the momentum-transfer region $q \lesssim 1.5$ F$^{-1}$, and use the factorizing-density approximation (for $0 \leqslant q^2 \leqslant 7.5$ F$^{-2}$ and for $A \geqslant 12$, the cross section error is $\lesssim 17\%$[1])

$$|\Psi_0(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A)|^2 = \prod_{j=1}^{A} \rho(\mathbf{r}_j). \qquad (1)$$

In (1), $\mathbf{r}_j$ is the radius vector of the $j$-th nucleon of the nucleus, $\Psi_0$ is the nuclear wave function, and $\rho$ is the single-particle density. The amplitude and the cross section for the elastic scattering of a nucleon by a nucleus are respectively

$$F = F_1 + F_2(\vec{\sigma}\vec{\nu}), \qquad (2)$$

$$\frac{d\sigma}{d\Omega} = |F_1|^2 + |F_2|^2 . \qquad (3)$$

In (2), $\sigma$ are Pauli matrices that act on the spin variables of the beam proton, $\nu = [\mathbf{q}\times\mathbf{1}]/q$, $\mathbf{1} = \mathbf{k}/k$, $\mathbf{q} = \mathbf{k} - \mathbf{k}'$, and $\mathbf{k}(\mathbf{k}')$ is the proton momentum before (after) collision with the nucleus ($\hbar = c = 1$). Using (1), we easily find that out of the five terms of the nucleon-nucleon ($NN$) amplitude, only amplitudes $a$ and $C(\boldsymbol{\sigma}\cdot\mathbf{n})$ contribute to the elastic process. We chose for them the following parametrization:

$$a = i a_0(1 - i\epsilon)\exp\{-\beta^2\Delta^2/2\}, \qquad (4)$$

$$C(\vec{\sigma}\mathbf{n}) = C_0(1 - i\epsilon)\Delta(\vec{\sigma}\mathbf{n})\exp\{-\beta^2\Delta^2/2\}, \qquad (5)$$

where $\boldsymbol{\Delta}$ is the momentum transferred to the $NN$ collision, $\mathbf{n} = [\boldsymbol{\Delta}\times\mathbf{1}]/\Delta$, $C_0$ is a complex constant, and $\epsilon$, $\beta^2$ and $a_0$ are real constants, with $a_0 = k\sigma_{NN}/4\pi$, where $\sigma_{NN}$ is the total $NN$-collision cross section. The use of (1) leads to $a = Za^{pp}/A + (1 - Z/A)a^{pn}$ and $C = ZC^{pp}/A + (1 - Z/A)C^{pn}$, where $a^{pp}$ ($a^{pn}$) is the amplitude of the elastic proton-proton (proton-neutron) scattering and $Z$ is the number of protons in the nucleus. The meanings of $C^{pp}$ and $C^{pn}$ are analogous. We define the functions $T$ and $t$ by the relations

$$T(b) = \frac{1}{2\pi ik}\int \exp\{-i\vec{\nabla}\mathbf{b}\}a(\Delta)S(\Delta)\,d^2\Delta, \qquad (6)$$

$$t(\mathbf{b}) = \frac{1}{2\pi ik}\int \exp\{-i\vec{\Delta}\mathbf{b}\}C(\Delta)(\vec{\sigma}\mathbf{n})S(\Delta)\,d^2\Delta, \qquad (7)$$

where the form factor $S$ is given by $S = \int \exp\{i\boldsymbol{\Delta}\cdot\mathbf{r}\}\rho(\mathbf{r})\,d^3r$. It follows then from (4)–(7) that ($\nabla_b \equiv \mathrm{grad}_b$)

$$t(\mathbf{b}) = C_0(\vec{\sigma}[\vec{\nabla}_b\times\mathbf{1}])\,T(b)/a_0 . \qquad (8)$$

The amplitude $F_1$ in the Glauber approximation[2] is equal, accurate to terms proportional to $C_0^2$, to the sum $F_1^{(0)} + F_1^{(2)}$, where

$$F_1^{(0)} = \frac{ik}{2\pi}\int \exp\{iq\mathbf{b}\}[1 - (1 - T)^A]\,d^2b, \qquad (9)$$

$$F_1^{(2)} = -\frac{ik}{2\pi}\,\frac{A(A-1)}{2}\,\frac{C_0^2}{a_0^2}\int \exp\{i\mathbf{q}\mathbf{b}\}(1 - T)^{A-2}(\nabla_b T)^2\,d^2b. \qquad (10)$$

In (10) we took into account (8) and the relation $(\sigma[\nabla_b T\times\mathbf{1}])^2 = (\nabla_b T)^2$. The amplitude $F_2$ is given by (we retain the term linear in $C_0$)

$$F_2(\vec{\sigma}\vec{\nu}) = \frac{ik}{2\pi}\int \exp\{i\mathbf{q}\mathbf{b}\}At(\mathbf{b})[1 - T(b)]^{A-1}\,d^2b. \qquad (11)$$

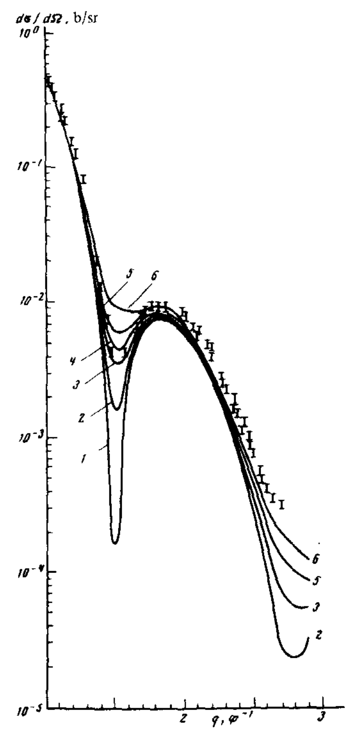


FIG. 1.

... of (6) in (11) and integration by parts yields

$$F_2 = -iC_0 q F_1^{(0)}/a_0 . \qquad (12)$$

It follows from (12) that the polarization in elastic scattering of the protons by nuclei (with the exception of the region of the diffraction minima, where $|F_1^{(2)}| \sim |F_1^{(0)}|$) is the same for nuclei with equal values of $Z/A$, since

$$P = 2\mathrm{Re}(F_1 F_2^*)/(|F_1|^2 + |F_2|^2) \approx \mathrm{Im}\{C_0 q/a_0\} . \qquad (13)$$

This fact was first noted in[3,4]. We now transform (10) with the aid of the identity ($\Delta_b \equiv \nabla_b^2$)

$$A(A-1)(1-T)^{A-2}(\nabla_b T)^2 = A(1-T)^{A-1}\Delta_b T - \Delta_b[1-(1-T)^A].$$

Integration in (10) by parts yields

$$F_1^{(2)} = -\frac{C_0^2 q^2}{2a_0^2} F_1^{(0)} - \frac{ik}{2\pi}\frac{C_0^2}{2a_0^2}\int \exp\{i\mathbf{qb}\} A(1-T)^{A-1}\Delta_b T d^2b. \qquad (14)$$

We denote the second term of (14) by the letter $\Phi$. The estimate of $\Phi$ is closely connected with allowance for the dependence of the phase of the $NN$ amplitude $\phi$ on $\Delta^2$.

We therefore consider, in place of (4), the more general expression

$$a = ia_0(1-i\epsilon)\exp\{-\beta^2\Delta^2/2 + \zeta\Delta^2\}, \quad \zeta = \eta + i\xi , \qquad (4a)$$

where $\phi \approx \xi\Delta^2$. For small $\zeta$, the change of $F_1$ is equal to

$$\delta F_1 = -\frac{ik}{2\pi}\zeta\int \exp\{i\mathbf{qb}\} A(1-T)^{A-1}\Delta_b T d^2b \qquad (15)$$

Comparison of (14) and (15) shows that both factors (the nucleon spin and the dependence of $\phi$ on $\Delta^2$) can be taken into account by introducing the effective parameter $\zeta_1(\zeta_1 = \eta_1 + i\xi_1)$, which is equal to $\zeta + C_0^2/(2a_0^2)$. It is seen from (4a) that introduction of $\eta_1$ is equivalent to a change of $\beta^2$ and leads to only a slight shift of the positions of the minima. Let us examine the effect of $\xi_1$ on $d\sigma/d\Omega$. Substituting (12), (14), and (15) in (3), we obtain

$$\frac{d\sigma}{d\Omega} = |F_1^{(0)} + \Phi_1|^2 + 2|F_1^{(0)}|^2\frac{q^2}{a_0^2}(\mathrm{Im}C_0)^2 - \mathrm{Re}\left\{\Phi_1^* F_1^{(0)}\frac{C_0^2 q^2}{a_0^2}\right\}, \qquad (16)$$

where $\Phi_1 = \delta F_1 + \Phi$. Equation (16) makes allowance for the fact that in the region of the minimum we have $|\Phi_1| \sim |F_1^{(0)}|$. The figure shows the dependence of the first term of (16) on $\xi_1$. The target nucleus is $^{12}C$, $E_{1ab} = 1.04$ GeV, while $\sigma_{NN}$, $\beta^2$, and $\epsilon$ in (4) and the oscillator parameter for $^{12}C$ were chosen equal to 44 mb, 0.2122 F$^2$, −0.275, and 0.401 F$^{-2}$, respectively.[5] Curves 1–6 correspond to $\xi_1$ equal to 0.15, 0, −0.1, −0.15, −0.2, and −0.3 F$^2$. According to[6], $\mathrm{Im}\{C_0^2/(2a_0^2)\} = 0.07$ F$^2$ at $E = 1$. As seen from the figure, the cross section decreases with increasing $\xi_1$, so that allowance for the nucleon spin only leads to a decrease of the cross section in the region of the minimum (the second and third terms of (16) cannot be offset by the decrease of the first, since $|C_0 q/a_0|^2 \lesssim 0.3$). It is possible to attain agreement with the experimental data[7] by assuming that the phase of the amplitude $a$ depends on $\Delta^2$, and it is seen from the figure that $-0.22$ F$^2 \leqslant \xi \leqslant -0.17$ F$^2$ ($\xi = -0.07 + \xi_1$). At higher energies, the contribution of $C(\boldsymbol{\sigma}\cdot\mathbf{n})$ to $d\sigma/d\Omega$ can be neglected completely, and only the dependence of $\phi$ on $\Delta^2$ needs to be taken into account. Thus, at $k = 2.1$ GeV/$c$ we have $\mathrm{Im}\{C_0/a_0\} \leqslant 0.1$ F.[8] Assuming that $|\mathrm{Re}\, C_0| \sim |\mathrm{Im}\, C_0|$, we obtain $|\mathrm{Im}\{C_0^2/(2a_0^2)\}| \sim 0.01$ F$^2$ and $|C_0 q/a_0|^2 \lesssim 0.04$.

We note that no allowance was made in this paper of the corrections that must be introduced in $d\sigma/d\Omega$ to account for the Coulomb scattering. This is perfectly permissible for $A \lesssim 20$. It was also assumed that the Fresnel and nonadiabatic corrections to $F_1$ and $F_2$ are small at $q \lesssim 1.5$ F$^{-1}$.

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