π^0-meson pole in γp scattering and proton polarizability

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It is shown that allowance for the π^0-meson pole is essential when the proton polarizability is determined from experiments on γp scattering. The sign of the $\pi^0 \rightarrow 2\gamma$ decay amplitude is obtained from experiments on γp scattering in the range 80–110 MeV, namely $g_{\pi NN}F_\pi < 0$.

The coefficients of the generalized electric $(\bar{\alpha})$ and magnetic $(\bar{\beta})$ proton polarizabilities were determined in[1] from experimental data on γp scattering in the energy range 80–110 MeV by using a theoretical expression derived for the differential cross section from general requirements in the form of a series in powers of the energy ν of the incident photon (in the l.s.), with terms up to ν^3 inclusive retained[2]

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \left(\frac{d\sigma}{d\Omega}\right)_p - \frac{e^2}{4\pi m}\nu^2[\bar{\alpha}(1+z^2) + 2\bar{\beta}z]\left(1 - 3\frac{\nu}{m}(1-z)\right) + 0(\nu^4), \tag{1}$$

where $z = \cos\theta$ and $(d\sigma/d\Omega)_p$ is the cross section for the scattering of photons by a structureless particle with spin 1/2.[3] It was assumed here that the contribution $O(\nu^4)$ of the discarded terms is ~2% in the investigated energy region.

From an analysis of γp scattering with the aid of the dispersion relations (DR)[4] it follows that the contribution of the π^0-meson pole (which enters in the differential cross section, starting with ν^4) is not small in the considered region of ν, and amounts to ~10% for ν ~100 MeV and $\theta = 150°$. Allowance for the π^0-meson pole may therefore turn out to be significant[5] when the values of $\bar{\alpha}$ and $\bar{\beta}$ are determined from experiment. In addition, this uncovers a possibility of determining the sign of the amplitude of the $\pi^0 \rightarrow 2\gamma$ decay from a comparison of theory with experiment on γp scattering at low energies.

In the present paper, on the basis of expression (1) for the differential cross section of γp scattering with allowance for the π^0-meson pole, we analyze the experimental data of[1] for the purpose of determining the coefficients $\bar{\alpha}$ and $\bar{\beta}$ of the generalized polarizability of the proton and of finding the relative sign of the $\pi^0 \rightarrow 2\gamma$ decay amplitude.

It is easy to show that the radius of the convergence of the expansion of the π^0-meson pole in powers of ν is equal to $\mu/\sqrt{2}$ for 90° scattering angle and to $\mu/2$ for 180°. Thus, this pole must be taken into account entirely in the investigated energy region, without expanding in ν.

The differential γp-scattering cross section with allowance for the π^0-meson pole is given by

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_0 + \frac{2}{m^2}\left(\frac{\nu}{\mu}\right)^2 \frac{1-z}{\left[1 + \frac{\nu}{m}(1-z)\right]^3} B_\pi(B_\pi + A) + 0(\nu^4), \tag{2}$$

where

$$B_\pi = \frac{\mu}{2\pi} g_{\pi NN} F_\pi \frac{t}{\mu^2 - t}, \qquad t = -2\nu^2(1-z)\left[1 + \frac{\nu}{m}(1-z)\right]^{-1},$$

$$A = \left(\frac{e^2}{4\pi}\right)\frac{\mu}{2m}\left\{(1+\lambda)^2 - 1 + (1+\lambda)(1-z)\left(1 + 2\frac{\nu}{m}\right)\left[1 + \frac{\nu}{m}(1-z)\right]^{-1}\right\}.$$

In these expressions, μ and m are the masses of the π^0 meson and proton, λ is the anomalous magnetic moment of the proton, $g_{\pi NN}$ is the constant for the π^0-meson interaction with the proton, $F_\pi = (64\pi^2\Gamma_{\pi^0 \rightarrow 2\gamma}/\mu^3)^{1/2}$ is the $\pi^0 \rightarrow 2\gamma$ decay amplitude, and $(d\sigma/d\Omega)_0$ is defined in (1) above.

In expression (2), the interference of the π^0-meson pole is taken into account only with the Born term. The contribution of the s-channel dispersion integral of the amplitude that contains the π^0-meson pole to the cross section starts with ν^4. This contribution can be calculated with the aid of the DR.[4] At $\nu = 100$ MeV it amounts to ~50% of the contribution of the π^0-meson pole (with the same sign as A). We assume henceforth that the contributions of the remaining discarded terms do not exceed the foregoing contribution, and that in the considered energy region the error due to the terms discarded in (2) is ±50% of the contribution of the π^0-meson pole. For a more accurate calculation of the uncertainty introduced into $\bar{\alpha}$ and $\bar{\beta}$ by the discarded terms it would be necessary to calculate the expressions (2) and the exact value of $d\sigma/d\Omega$ with the aid of the DR and to find their difference.

We shall use formula (2) to analyze the experimental data of[1] separately for $\theta = 90°$ and $\theta = 150°$. From an analysis for $\theta = 90°$ we obtain $\bar{\alpha}$, and the analysis for $\theta = 150°$ makes it possible to determine directly the difference $\bar{\alpha} - \bar{\beta}$. The results are listed in the table for different manners of taking the π^0-meson pole into account. The width of the $\pi^0 \rightarrow 2\gamma$ decay is taken from[5] to be $\Gamma_{\pi^0 \rightarrow 2\gamma} = 7.7 \pm 0.9$ eV. The table gives the probabilities $P(\chi^2)$ for three different assumptions concerning the

	$g_{\pi NN}F_\pi < 0$	$g_{\pi NN}F_\pi = 0$	$g_{\pi NN}F_\pi > 0$	Δ
$\bar{\alpha} \cdot 10^{43}$ cm^3	13.9	10.7	8.0	± 2.1
$P_{90°}$	9%	25%	50%	–
$(\bar{\alpha} - \bar{\beta}) \cdot 10^{43}$ cm^3	19.9	11.4	4.4	± 4.4
$P_{150°}$	12%	1.3%	0 7%	–

meson pole. The last column of the table gives the summary experimental and (proposed) theoretical errors of $\overline{\alpha}$ and $\overline{\alpha} - \overline{\beta}$.

It is seen from the table that the polarizability difference $\overline{\alpha} - \overline{\beta}$ is particularly sensitive to allowance for the contribution of the π^0-meson pole.

In addition, it follows from the table that an analysis of the data for the angle 90° does not make it possible to choose any of the signs of F_π. This is connected with the fact that the contribution of the π^0-meson pole for $\theta = 90°$ is not large in the investigated energy region. On the other hand, in the analysis of the experimental data at $\theta = 150°$ one can separate reliably (the probabilities $P(\chi^2)$ for the competing hypotheses are 1.3 and 0.7%) the sign of $g_{\pi NN} F_\pi < 0$. The obtained sign of F_π coincides with the sign obtained in[7,4] for an analysis of γp scattering with the aid of the DR, and with the sign obtained in perturbation theory by considering the $\pi^0 \to 2\gamma$ decay via a nucleon-antinucleon pair.[8]

Thus, an investigation of low-energy γp scattering with the aid of expression (2) makes it possible, generally speaking, to determine by a model-independent method the sign of the $\pi^0 \to 2\gamma$ decay amplitude, provided that a sufficiently reliable estimate is made of the terms discarded in (2).

The obtained value $\overline{\alpha} - \overline{\beta} = (19.9 \pm 4.4) \times 10^{-43}$ cm^3 agrees with the predictions of the sum rules[5] ($\overline{\alpha} - \overline{\beta} = 19.2 \times 10^{-43}$ cm^3) and indicates that it is insufficient to take the t-channel contribution in γp scattering into account only with the aid of fourth-order diagrams of perturbation theory, and there is apparently justification for taking into account other states within the framework of the bootstrap model, using the Pade approximations for the summation of the partial waves.

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