On the possibility of Cerenkov emission of γ quanta by electrons

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(Submitted November 22, 1975)
Pis'ma Zh. Eksp. Teor. Fiz. **23**, No. 1, 34–36 (5 January 1976)

It is shown that when account is taken of the resonant scattering of γ quanta by nuclei the refractive index for them can become larger than unity, so that monochromatic Cerenkov γ radiation from fast electrons becomes possible. Estimate of its intensity are given.

PACS numbers: 78.70.–g

1. It is known that the reason why there is no Cerenkov radiation in the x-ray region is that the x-ray frequencies are higher than all the resonant frequencies of the atom, so that the refractive index obtained for x rays as a result of Rayleigh scattering by atoms is always smaller than unity.

2. The situation changes if account is taken of coherent scattering of photons by nuclei. Then the negative increment to unity in the refractive index, due to the Rayleigh amplitude, can be offset by a positive contribution from the amplitude of resonant γ-ray scattering by the nucleus. As a result the refractive index can become larger than unity for γ rays with energy close to resonant. Therefore when an electron moves in a medium containing nuclei that scatter γ rays resonantly, Cerenkov emission of γ rays can occur near the resonant frequency when the refractive index exceeds unity.

3. It is known that the refractive index for photons is connected with the forward scattering amplitude f by (see[1])

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$$n = 1 + \frac{\lambda^2}{2\pi} Nf \equiv 1 + F, \tag{1}$$

where λ is the photon wavelength and N is the number of atoms per unit volume. The forward scattering amplitude f, when nuclear scattering is taken into account, is the sum of the Rayleigh amplitude f^R and of the coherent nuclear-scattering amplitude f^N:

$$f = f^R + f^N \quad . \tag{2}$$

The Rayleigh amplitude is determined near the absorption edges by the classical electron radius r_e

$$f^R = -Zr_e, \tag{3}$$

where Z is the atomic number. The coherent nuclear forward amplitude for dipole transition is given by (see, e.g., [2])

$$f^N = -\frac{2j'+1}{2j+1} \frac{\lambda}{8\pi} \frac{f^2(k^2)\Gamma_i}{\omega - \omega_0 + (i\Gamma/2)}, \tag{4}$$

where Γ_i is the radiative width, Γ is the total width of the level and is connected with Γ_i by the relation $\Gamma = (1+\alpha)\Gamma_i$, α is the internal-conversion coefficient, $f(k^2)$ is the Lamb-Mössbauer factor, j' and j are respectively the spins of the excited and ground states of the nucleus, ω is the γ-quantum frequency, and ω_0 is the frequency of the resonant level. The real part of the amplitude f_r, which determines the real part of the refractive index n_r, is of the form[1)]

$$f_r = -Zr_e - \frac{2j'+1}{2j+1} \frac{\lambda}{8\pi} \frac{\Gamma_i \Delta}{\Delta^2 + \Gamma^2/4}, \tag{5}$$

where $\Delta = \omega - \omega_0$. It is easily seen from (5) that f_r has a maximum at $\Delta = -\Gamma/2$, with

$$\max f_r = -Zr_e + \frac{2j'+1}{2j+1} \frac{\lambda}{8\pi} \frac{\Gamma_i}{\Gamma} \quad . \tag{6}$$

Thus, for γ-quantum wavelengths $\lambda > 8\pi Zr_e$ the real part of the refractive index can exceed unity in a certain region to the left of the resonance (at $\omega < \omega_0$). Therefore when a charged particle moves in such a medium with velocity $v > c/n_r$, Cerenkov radiation is produced at a frequency $\omega \approx \omega - \Gamma/2$.

4. To estimate the radiation intensity we can use the Budini formula[3] for the number of photons emitted per unit time when a charged particle moves in a medium with a complex refractive index. At $|n_r - 1| \ll 1$ we have

$$\frac{dN}{dt} = \frac{e^2}{\hbar c} \int \left[1 - \frac{1}{\beta^2 n_r^2(\omega)}\right] d\omega, \tag{7}$$

where $\beta = v/c$, and the integration is over the region $\beta n_r > 1$. Using (1) and the fact that $\beta^2 = 1 - (1/\epsilon^2)$, where $\epsilon = mc^2/m_0c^2$ is the total energy of the particle in units of rest mass, we obtain

$$\frac{dN}{dt} = \frac{e^2}{\hbar c} \int \left[F_r(\omega) - \frac{1}{\epsilon^2}\right] d\omega \quad . \tag{8}$$

Integrating (8), we obtain ultimately

$$\frac{dN}{dt} = \frac{e^2}{\hbar c}\left(\frac{1}{\epsilon^2} - F^R\right)\frac{\Gamma}{2}\left[\gamma \ln\frac{\delta_-^2 + 1}{\delta_+^2 + 1} - (\delta_+ - \delta_-)\right], \tag{9}$$

where

$$\gamma = \max F_\gamma^N / [(1/\epsilon^2) - F^R], \quad \delta_\pm = -\gamma \pm \sqrt{\gamma^2 - 1}.$$

5. By way of example we consider the isotope ^{73}Ge with the Mössbauer transition $7/2^+ \rightarrow 9/2^+$ (transition energy ~67 keV). For this isotope $\alpha = 0.2$ and $\Gamma \approx 10^8$ Hz.[4] Using (3) and (5) we have $|F^R| \approx 3\times10^{-7}$, and max $F_\gamma^N \approx 18\times10^{-7}$. That is to say, the maximum of the nuclear amplitude is six times the Rayleigh amplitude. For an electron energy ~3 GeV ($\epsilon \approx 6\times10^3$) we have $\gamma = 6$, $\delta_+ \approx -0.1$, and $\delta_- \approx -11.9$, so that the number of the photons radiated by the electron per unit time turns out to be $dN/dt \approx 4$ sec^{-1}. At a current ~10 μA, from a target of thickness ~10^{-3} cm (the maximum γ-ray absorption length at resonance is ~10^{-3} cm) we have ~10 photons/sec. If an electron storage ring is used, this quantity is increased by at least three orders of magnitude, i.e., to ~10^4 photons/sec. We note that photons can be emitted in practice forward at an angle $\theta = \sqrt{2F_\gamma - 1/\epsilon^2} \lesssim 10^{-3}$ to the direction of electron motion. The energies of the emitted photons will lie in a very narrow interval on the order of several line widths, amounting to ~10^{-6} eV while the energy of the γ quanta themselves is ~67 keV.

In conclusion, the authors are deeply grateful to Yu.S. Grushko, V.A. Ruban, and O.I. Sumbaev for useful discussions.

1)The Lamb-Mössbauer factor is assumed equal to unity.

[1]R.G. Newton, Scattering Theory of Waves and Particles, McGraw, 1966.
[2]V.A. Belyakov, Usp. Fiz. Nauk **115**, 553 (1975) [Sov. Phys. Usp. **18**, 267 (1975)].
[3]P. Budini, Nuovo Cimento **10**, 236 (1953).
[4]Mössbauer Effect Data Index, 1969, ed. J.G. Stevens and V.E. Stevens, IFE/Plenum, 1969.