Resonance effects in mesic atoms

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We consider the effect of resonant excitation of nuclear levels in mesic atoms.

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1. If the energy of a nuclear transition in a mesic atom is close to the energy difference of two mesic-atom states, resonant excitation of the nuclear levels can occur. Effects of this kind should be expected both in μ-mesic atoms, where the resonance is due to electromagnetic interaction, and in π and K atoms, where the strong interaction must also be taken into account.[1]

ance can really be observed only if the matrix element $V^{(\lambda)}$ of the interaction that causes the resonance is of the order of or larger than v difference between the mesic and nuclear transitions.

ase of resonance, the probability of excitation of a nuclear level de- on the widths Γ_1 and Γ_2 of the mesic-atom levels. If these widths enough ($\Gamma_1, \Gamma_2 \gtrsim V_{ij}^{\lambda}$) then a solution of the nonstationary Schrödinger ıust be obtained in order to calculate the excitation probabilities of -atom and nuclear levels. It is convenient to seek this solution in the form given in[2].

2. We consider two mesic-atom levels with energies E_1 and E_2. The meson can undergo a dipole transition from these levels to levels E_1^0, and $E_{2'}^0$, respectively.

Assuming that at the initial instant the level E_1 is populated with unity probability, we write down the system of Heitler's equations in the form

$$G_1 = 1 + V^{(\lambda)}_{1;2,N} \frac{1}{E - E_2 - E_N + i\epsilon} G_2 + \sum_\nu H^{\gamma}_{1;1'}(\nu) \frac{1}{E - E_{1'} - E_\nu + i\epsilon} G_{1'}(\nu),$$

$$G_2 = V^{(\lambda)}_{2,N;1} \frac{1}{E - E_1 + i\epsilon} G_1 + \sum_\nu H^{\gamma}_{2N;2'}(\nu) \frac{1}{E - E_{2'} - E_\nu - E_N + i\epsilon} G_{2'}(\nu), \tag{1}$$

$$G_{1'}(\nu) = H^{\gamma}_{1',1}(\nu) \frac{1}{E - E_1 + i\epsilon} G_1,$$

$$G_{2'}(\nu) = H^{\gamma}_{2',N;2,N} \frac{1}{E - E_2 - E_N + i\epsilon} G_2,$$

$$\epsilon \to +0.$$

Here G_i is the probability amplitude for the corresponding states, the index γ pertains to the γ quantum, $H^{\gamma}_{i,j}$ is the $E1$ matrix element for the emission of a γ quantum, $V^{(\lambda)}_{i,j}$ is the matrix element of the meson-nucleus interaction operator, and E_N is the energy of the excited nuclear level for which the resonance condition $E_N \approx E_1 - E_2$ is satisfied. The solution of (1) is obtained in a manner similar to that of[3]:

$$G_2 = V^{(\lambda)}_{2,N;1}(E - E_2 - E_N)\left[\left(E - E_2 - E_N + i\frac{\Gamma_2}{2}\right)\left(E - E_1 + i\frac{\Gamma_1}{2}\right) - \left|V^{(\lambda)}_{1;2,N}\right|^2\right]^{-1}. \tag{2}$$

The probability of the transition $2 \to 2'$, which is determined by the amplitude $G_{2'}(\nu)$ at $E = E_\nu + E_{2'} + E_N$, is equal to[2]

$$W_\gamma(2 \to 2') = \sum_\nu \left| b_{2 \to 2'}(\nu) \right|^2, \tag{3}$$

where

$$b_{2\to 2'}(\nu) = V^{(\lambda)}_{2,N;1} H^{\gamma}_{2',N;2,N}\left[\left(E_{2'} + E_\nu - E_2 + i\frac{\Gamma_2}{2}\right)\left(E_{2'} + E_\nu + E_N - E_1 + i\frac{\Gamma_1}{2}\right) - \left|V^{(\lambda)}_{2,N;1}\right|^2\right]^{-1}. \tag{4}$$

Replacing the summation in (3) by integration, expanding the denominator of the integrand in terms of the poles, and using the residue theorem, we obtain

$$W_\gamma(2\to 2') = \frac{1}{4}\left|V^{(\lambda)}_{2,N;1}\right|^2\Gamma_2(\Gamma_1+\Gamma_2)\left\{\left[4a^2\eta + \frac{1}{4}(\Gamma_1+\Gamma_2)^2\right] \times\left[\frac{1}{16}(\Gamma_1+\Gamma_2)^2 - a^2(1-\eta)\right]\right\}^{-1}. \tag{5}$$

We use here the notation

$$\eta = \frac{1}{2}(1+\delta), \qquad \delta = (1+x^2)^{\mp 1/2},$$

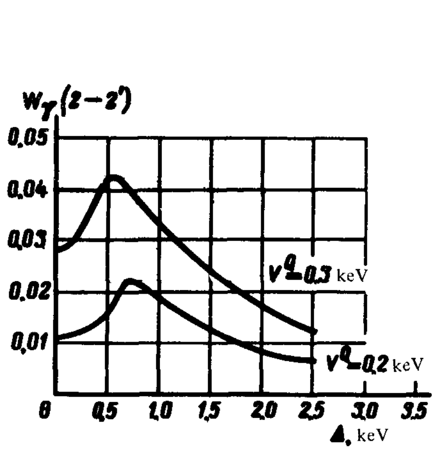
$$x = \frac{1}{4}\Delta(\Gamma_1 - \Gamma_2)\left[\frac{1}{4}\Delta^2 + \left|V^{(\lambda)}_{2,N;1}\right|^2 - \frac{1}{16}(\Gamma_1 - \Gamma_2)^2\right]^{-1}, \tag{6}$$

$$a = \left\{\left[\frac{1}{4}\Delta^2 + \left|V^{(\lambda)}_{2,N;1}\right|^2 - \frac{1}{16}(\Gamma_1-\Gamma_2)^2\right]^2 + \frac{1}{16}\Delta^2(\Gamma_1-\Gamma_2)^2\right\}^{1/4},$$

$$\Delta = E_2 + E_N - E_1.$$

If the relations $\Gamma_1,\ \Gamma_2 \ll |V^{(\lambda)}_{ij}|$ are satisfied, formula (5) becomes much simpler:

$$W_\gamma(2\to 2') = \left|V^{(\lambda)}_{2,N;1}\right|^2\Gamma_2(\Gamma_1+\Gamma_2)\left[\Gamma_1\Gamma_2\Delta^2 + (\Gamma_1+\Gamma_2)^2\left|V^{(\lambda)}_{2,N;1}\right|^2\right]^{-1}. \tag{7}$$



Expression (7) can also be obtained by using the solution of the stationary Schrödinger equation.

3. By way of example we consider the ^{238}U mesic atom. In this case the difference of the energy levels $4f_{7/2}$ and $4p_{3/2}$ turns out to be close to 44.5 keV, which coincides almost exactly with the excitation energy E_N of the rotational level 2^+. Thus, in this case we have a system of two transtions that are at resonance and, in the presence of even a weak interaction a certain admixture of the state $4p_{3/2}$ appears in the wave function of the system. This leads to the appearance of additional transitions ($4p_{3/2} \rightarrow 3s_{1/2}$), which can be observed in experiment. Calculations yielded for the quadrupole interaction $V^{(Q)} = 0.25$ keV, and the widths $\Gamma_1 = 1.8$ and $\Gamma_2 = 0.2$ keV. The dependence of $W_\gamma(2 \rightarrow 2')$ on Δ is shown in the figure, from which it is seen that the probability of the transition $4p_{3/2} \rightarrow 3s_{1/2}$ can reach $\sim 3\%$.

The proposed calculation method can be used also for π and K mesic atoms,[4] where the widths of the corresponding levels are as a rule much larger than for μ mesic atoms. We note that an analysis similar to the foregoing one was made in[5] for the case of a continuous spectrum.

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