Concerning the form factors of composite systems

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(Submitted November 9, 1975)
Pis'ma Zh. Eksp. Teor. Fiz. **23**, No. 1, 58–61 (5 January 1976)

Problems involved in the asymptotic behavior of relativistic form factors of composite system are considered. Arguments are advanced indicating a possible influence of the structure of the nucleons on the behavior of the form factors of atomic nuclei.

PACS numbers: 21.10.—k, 13.40.Fn

The asymptotic behavior of the form factors of composite systems has recently been the subject of intense theoretical and experimental research (see e.g., [1–6]). The connection $n = N - 1$ of the exponent n in the asymptotic behavior of the electromagnetic form factors $F(t) \sim |t|^{-n}$, N being the number of elementary pointlike components contained in the hadron, obtained within the framework of dimensional analysis,[7,8] urges a search for a dynamic explanation of this relation.

This article deals with the asymptotic behavior of the form factors from the point of view of the dynamic quasipotential equations in terms of the "light-front" variables.[9,10]

The expressions for the electromagnetic form factors of two- and many-particle systems, in terms of wave functions $\Phi(x, \mathbf{p}_\perp)$ and $\Phi([x^{(i)}, \mathbf{p}_\perp^{(i)}])$ satisfying the equations

$$\left[P^2 - \frac{m^2 + p_\perp^2}{x(1-x)}\right]\Phi(x, p_\perp) = \int_0^1 \frac{dx'}{x'(1-x')} \int dp_\perp' V(P; x, p_\perp; x', p_\perp')\Phi(x', p_\perp') \tag{1}$$

$$\left[P^2 - \sum_{i=1}^{N} \frac{m^{(i)2} + p_\perp^{(i)2}}{x^{(i)}}\right] \Phi([x^{(i)}, p_\perp^{(i)}])$$

$$= \int_0^1 \prod_{i=1}^{N} dx^{(i)'} (x^{(i)'})^{-1} \delta(1 - \sum_{i=1}^{N} x^{(i)'}) \int \prod_{i=1}^{N} dp_\perp^{(i)'} \delta^{(2)}(\sum_{i=1}^{N} p_\perp^{(i)})$$

$$\times V(P; [x^{(i)} p_\perp^{(i)}]; [x^{(i)'}, p_\perp^{(i)'}]) \Phi([x^{(i)'}, p_\perp^{(i)'}]), \tag{2}$$

take respectively the form

$$F_2(t) = 2(2\pi)^3 \int_0^1 \frac{dx}{x(1-x)} \int dp_\perp \Phi(x, p_\perp + (1-x)\vec{\Delta}_\perp) \Phi(x, p_\perp), \tag{3}$$

$$F_N(t) = (2\pi)^4 (2i)^{-1-N} (2\pi i)^{1-N} \int_0^1 \prod_{i=1}^{N} dx^{(i)} (x^{(i)})^{-1} \delta(1 - \sum_{i=1}^{N} x^{(i)})$$

$$\times \int \prod_{i=1}^{N} dp_\perp^{(i)} \delta^{(2)}(\sum_{i=1}^{N} p_\perp^{(i)}) \Phi(x^{(1)}, p_\perp^{(1)} + (1 - x^{(1)})\vec{\Delta}_\perp, [x^{(i)}, p_\perp^{(i)} - x^{(i)}\vec{\Delta}_\perp]_{i \neq 1})$$

$$\times \Phi([x^{(i)}, p_\perp^{(i)}]); \quad t = \Delta^2 = -\vec{\Delta}_\perp^2. \tag{4}$$

For simplicity, we consider here the case when one (in this case the first) particle is charged.

In formulas (1)—(4) the gauge-invariant variables $x^{(i)}$ have been introduced in accord with the formula

$$x^{(i)} = p_+^{(i)} / P_+ = (p_0^{(i)} + p_3^{(i)}) / (P_0 + P_3), \tag{5}$$

where $p^{(i)}(p_+, p_-, p_\perp)$ is the 4-momentum of the ith particle, and $P(P_+, P_-, P_\perp = 0)$ is the 4-momentum of the composite system. We note that a formula of the type (3) is obtained also in the framework of the parton model in a system with infinite momentum.[11]

We obtain the principal terms of the asymptotic expressions (3) and (4) for the form factors accurate to the possible logarithmic factors, assuming a definite behavior of the paired interactions in (1) and (2).

For the sake of clarity, we write down the quasipotential of the paired interactions in the case of a three-particle system

$$V(P;[x^{(i)},\ \mathbf{p}_\perp^{(i)}];[x^{(i)\prime},\ \mathbf{p}_\perp^{(i)\prime}]) = \sum_{i=1}^{3} p_+^{(i)}\delta(p_+^{(i)} - p_+^{(i)\prime})\delta^{(2)}(\mathbf{p}_\perp^{(i)} - \mathbf{p}_\perp^{(i)\prime})$$

$$\times V_i^{(2)}\left(P_- - \frac{m^{(i)2} + \mathbf{p}_\perp^{(i)2}}{p_+^{(i)}},\ P_+^{(jk)}, \mathbf{P}_\perp^{(jk)};\ x^{(j)},\ \mathbf{p}_\perp^{(j)},\ x^{(k)};\ \mathbf{p}_\perp^{(k)};\ x^{(j)\prime},\ \mathbf{p}_\perp^{(j)\prime},\ x^{(k)\prime};\ \mathbf{p}_\perp^{(k)\prime}\right). \quad (6)$$

Here $V_i^{(2)}$ are the quasipotentials of the interactions of the two-particle subsystems (jk), $P^{(jk)}$ are the total momenta of the two-particle subsystems. Questions of the theory of scattering of many-particle systems in the spirit of Faddeev's N-particle equations[12] will be considered separately.

According to (3) and (4), the behavior of the form factors at large momentum transfers is determined by the asymptotic forms of the wave functions at large values of the transverse variables $|p_\perp^{(i)}|$. Putting

$$V^{(2)}\Big|_{|\mathbf{p}_\perp| \to \infty} \sim 1/|\mathbf{p}_\perp^2|^{\theta}, \quad (7)$$

we obtain the following asymptotic estimates:

$$F_2(-\vec{\Delta}_\perp^2)\Big|_{|\Delta_\perp^2| \to \infty} \sim V^{(2)}/\vec{\Delta}_\perp^2 \ , \quad (8)$$

$$F_N(-\vec{\Delta}_\perp^2)\Big|_{|\vec{\Delta}_\perp^2| \to \infty} \sim (V^{(2)}/\vec{\Delta}_\perp^2)^{N-1} . \quad (9)$$

In formation on the field-theoretical model corresponding to a potential of the type (7) is contained in the exponent θ. In particular, exchange of a scalar particle corresponds to $\theta = 1$. The results of the dimensional analysis[7,8] are reproduced in the limiting case $\theta \to 0$. It should be noted that the results (8) and (9), as well as the results of the dimensional analysis, pertain to the case when the elementary components (quarks or partons), which enter in the hadron, are themselves pointlike.

When an attempt is made to extend these results to the case of atomic nuclei, the question arises of the possible manifestation of the fact that the nucleons inside the nucleus are not pointlike. It is apparently necessary in this case to assume that the rate of decrease of the form factor will be larger than prescribed by (8) and (9). For a more definite answer to this question it is necessary, in our opinion, to resort to information on the nuclear wave functions from experiments with beams of relativistic nuclei (see, e.g., [13]), to construct the corresponding form factors assuming that the nucleons are pointlike, and compare the results with direct measurements of the form factors of the nuclei. Differences (agreement) between the results of these two analysis will serve as arguments for (against) the assumption that the fact that the nucleons in the nucleus are not pointlike is significant. In any case, this raises the interesting problem of the relativistic description of systems of extended (nonpointlike) particles. In this case considerations of geometric character may be quite useful here (see[14] in this connection). We shall return in the future to a detailed treatment of these questions.

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