Wave functions of relativistic nucleons in nuclei

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Invariant wave functions are introduced for the description of nuclei at relativistic nucleon momenta. It is shown that deviation from spherical symmetry appears in the wave functions at small distances.

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The formalism of invariant wave functions (WF), which describe nuclei correctly at nucleon momenta on the order of their masses, is presently vital in nuclear physics. The momentum region $q \sim m$ has already been "sounded" experimentally; thus, the deuteron form factor is known up to $q^2 \lesssim 6$ (GeV/c)^2.[2] In this region, the description of the aggregates of the experimental data with the aid of nonrelativistic wave functions $\Psi(\mathbf{q})$ suitably fitted at $q \sim m$ is doomed to failure, since the very parameterization of the WF can change (and indeed does change), and does not reduce to a dependence of only one argument—the relative momentum $\mathbf{q}$. To describe different experiments it would be necessary

to have different wave functions $\Psi(q)$. The problem of finding relativistic WF was posed and considered in[2] in the coordinate representation.

The purpose of this article is to report results of an investigation of relativistically-invariant WF needed for the description of nuclei at relativistic values of the nucleon momenta. These wave functions can be useful also in composite models of elementary particles. Details will be published in a more extensive article. Preliminary results were published in the review.[3]

A relativistic bound system containing, generally speaking, an indeterminate number of particles, is described by a state vector $\phi(p)$. Its expansion in states with a fixed number of particles is given by

$$\phi(p) = \sum_n \int C_n(\mathbf{k}_1, \ldots, \mathbf{k}_n, p) a^+(\mathbf{k}_1) \ldots a^+(\mathbf{k}_n)| 0 >$$

$$\times \delta^{(3)}(\mathbf{k}_1 + \ldots + \mathbf{k}_n - \mathbf{p}) \frac{d^3 k_1}{\sqrt{2\epsilon_1}} \ldots \frac{d^3 k_n}{\sqrt{2\epsilon_n}} , \tag{1}$$

where C_n are components of the Fock column, and all the momenta lie on the mass shells: $k_i^2 = m^2$, $p^2 = M^2$. We consider the case of spinless particles, as well as bound systems with zero total angular momentum.

Under transformations of the Poincaré group, $x \to x' = x + \delta x$ and $\delta x_i = \delta_i + \delta\omega_{ik} x_k$ the state vector $\phi(p)$ is transformed in the following manner: $\phi(p) \to \phi'(p') = \hat{U}\phi(p) = (1 + \delta\hat{U})\phi(p)$, where $\delta\hat{U} = iP_k \delta\epsilon_k + (i/2)M_{ik}\delta\omega_{ik}$, while P_k and M_{ik} are generators of the Poincaré group. Bearing in mind that P_k transforms in accord with $P'_k = \hat{U}^{-1} P_k \hat{U} = P_k - \delta\omega_{ik} P_i$, we obtain

$$\phi(p_k - \delta\omega_{ik} p_i) = (1 + \delta\hat{U})\phi(p_k). \tag{2}$$

In the nonrelativistic case, for example, for a two-particle system, Eq. (2) leads to Galilean invariance of the WF, namely $C_2(\mathbf{k}_1 - m\delta\mathbf{v}, \mathbf{k}_2 - m\delta\mathbf{v}, \mathbf{p} - M\delta\mathbf{v}) = C_2(\mathbf{k}_1, \mathbf{k}_2, \mathbf{p})$. Eliminating $\mathbf{k}_2$, we see that the WF depends only on one argument $\mathbf{q} = \mathbf{k}_1 - (m/M)\mathbf{p}$. In the relativistic case the generators of the Lorentz transformations contain an interaction Hamiltonian that changes the number of particles. Therefore the WF are transformed not only via themselves, but also via other components. Thus, for a separately taken component C_2 there is no group transformation law, this component is not invariant and depends (after eliminating $\mathbf{k}_2$) on two vector arguments separately: $C_2 = C_2(\mathbf{k}_1, \mathbf{p})$. Some simplification can be obtained by going over to the light cone (in a system with infinite momentum),[4] where the dependence on the modulus of $\mathbf{p}$ drops out, but the dependence on the direction $\mathbf{p}/|\mathbf{p}|_{\mathbf{p}\to\infty}$ remains. Thus, the relativistic WF contain one "excess" variable in comparison with the nonrelativistic ones, and this variable is the same in all the components of the Fock column. It is of the form of a three-dimensional unit vector and characterizes the connection between components that leads to covariance of the state vector.

The noninvariance of the Fock component is an exceedingly inconvenient property of the theory. Our main task is to impart to the theory an invariant form that makes it possible to parametrize the WF conveniently and to make the entire formalism "profitable." To this end we consider more general WF, which go over in a particular case to Fock components. Instead of the two-particle component we introduce

$$C = C(k_1, k_2, p, \lambda r), \tag{3}$$

where λ is a 4-vector such that $\lambda^2 = 1$ and $\lambda_0 > 0$, τ is a scalar parameter, $k_1^2 = k_2^2 = m^2$, $p^2 = M^2$, and the following equality holds:

$$k_1 + k_2 = p + \lambda r.$$

We shall call the 4-momentum $\lambda \tau$ the spurion momentum. If $\vec{\lambda} = 0$ and $\lambda_0 = 1$, then the parameter τ can be eliminated: $\tau = \epsilon(k_1) + \epsilon(k_2) - \epsilon(p)$, and the function (3) is chosen such as to go over after this to the Fock component $C_2(k_1, k_2, p)$. All the other components are analogously generalized.

The method of generalizing (3) is not unique. In order for it not to distort the functional dependence of the WF and, conversely, emphasize it, it is necessary that the method correspond to a definite dynamic scheme. We introduce the functions (3) in such a way that they correspond to the three-dimensional field-theory formulation proposed by Kadyshevskiĭ.[5] It is convenient for our purposes, in particular, because of the exceeding simplicity of the final result—the parametrization of the WF. We note also that this formulation is quite general and is based on an equation of the Tomonaga-Schwinger type, and that at present there is no alternative whatever.

The transition to the light cone is effected by the substitution $\lambda \to \omega$, where $\omega = (\omega_0, \vec{\omega})$, $\omega^2 = 0$, and $\omega_0 > 0$.

We introduce the variables

$$q_1 = k_1 (-) \frac{m}{\sqrt{Q^2}} Q = k_1 - \frac{Q}{\sqrt{Q^2}} \left[\epsilon(k_1) - \frac{(k_1 Q)}{\sqrt{Q^2} + Q_0} \right] ,$$

$$q_2 = k_2 (-) \frac{m}{\sqrt{Q^2}} Q = -q_1 ,$$

$$\tag{4}$$

$$\vec{\omega}' = \vec{\omega} - \frac{Q}{\sqrt{Q^2}} \left[\omega_0 - \frac{(\vec{\omega} Q)}{\sqrt{Q^2} + Q_0} \right] ,$$

$$n = \vec{\omega}' / |\vec{\omega}'| ,$$

where $Q = p + \omega \tau$.

It is easily seen that the WF (3) depends only on two independent vector arguments, $q \equiv q_1$ and n:

$$C = C(q, n). \tag{5}$$

Under the Lorentz transformation, the vectors q and n experience only rotation, so that the WF is invariant.

The appearance of the variable n corresponds precisely to the dependence on the direction $p/|p|$. We note that introduction of the variables q and n solves automatically the problem of separating the motion of the mass center, in analogy with the variable $q = k - (m/M)p$ in the nonrelativistic case.

The variables q and n are connected with the known variables in the a system with infinite momentum $q_\perp$ and x:

$$q_\perp^2 = q^2 - (n q)^2,$$
$$x = \frac{1}{2}\left(1 - \frac{(n q)}{\epsilon(q)}\right). \tag{6}$$

According to[6] it is possible to introduce a coordinate space by using the Shapiro transformation[7] with respect to the variable q. It is Fourier-conjugate to the rapidity space. [2] The WF (3) are connected in simple fashion with the vertex parts $\Gamma(k_1, k_2, p, \omega\tau)$ of the diagram technique[5]:

$$C(k_1, k_2, p, \omega r) = \frac{1}{2\pi(s - M^2)}\Gamma(k_1, k_2, p, \omega r), \tag{7}$$

where $s = (k_1 + k_2)^2$.

The appearance of a vector **n** in formula (5) can be interpreted as the onset of a certain deviation from the sphericity of the WF at short distances. In the nonrelativistic case the operator δU in (2) generates Galilean transformations, and at $q \ll m$ the dependence on **n**, and with it the nonsphericity, vanishes: $C(q, n)|_{q \ll m} \to \Psi(q)$.

We have thus obtained an invariant formalism of WF that depends on three-dimensional arguments, have a probabilistic interpretation, and have a representation in relativistic coordinate space. It follows from this formalism that to relativize, say, the WF of the deuteron it is necessary, besides introducing other components (isobaric or pionic) to change the parametrization of the WF by introducing in them the additional argument **n.**

In the case of an arbitrary number of particles, the prescription for the generalization is the same: $\Psi_n(q_1, \ldots, q_{n-1}) \xrightarrow{q \gg m} C(q_1, \ldots, q_{n-1}, n)$.

After this paper was completed, the author learned of a paper sent to press by M. V. Terent'ev, "Structure of the Wave Functions of Mesons as Bound States of Relativistic Quarks," devoted to a study of the WF of a system consisting of only two relativistic particles. In this case the WF depends only on one argument, **q.**

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