Spinor gauge superfield

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A supersymmetrical gauge theory with the most general localization of the internal symmetries in superspace is proposed. The spinor gauge fields enters in the Lagrangian polynomially and includes a field with spin 3/2.

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1. Wess, Zumino, and Ferrara[1,2] and Salam and Strathdee[3,4] have proposed supersymmetrical gauge theories in which the parameters of the transformations of the internal symmetries are scalar superfunctions of limited type, namely *chiral scalar superfunctions* $\lambda_i(x,\theta)$, where θ are spinor anticommuting coordinates. They include in an essentially nonlinear fashion a real scalar gauge superfield that contains a gauge vector Yang-Mills field and a spinor gauge field. Such theories can be called chiral gauges supersymmetric theories (CGST).[1)]

In this paper we propose a general gauge supersymmetric theory (GGST), in which the internal symmetry is realized locally in the superspace (x,θ) in the most general manner. The transformation parameters are scalar functions of general type, not only chiral. The role of the gauge superfield is played by a *spinor* Majorana superfield that includes the Yang-Mills vector field and a *field with spin* 3/2. The Lagrangian and the equations of motion of the GGST contain the gauge superfield polynomially.

2. We consider the set of superfields $V_m(x,\theta)$ that transform in accordance with a certain representation of the internal symmetries with generators $(T_i)_{mn}$

$$V'_m(x,\theta) = [\exp(i\lambda_i T_i)]_{mn} V_n(x,\theta). \tag{1}$$

The invariant Lagrangian for these free superfields includes the fields themselves and the differential operators made up of the spinor derivatives $D_\alpha = \partial/\partial\bar\theta^\alpha - (i/2)(\not\partial\theta)_\alpha$.[4,9] [2)]

In complete analogy with the Yang-Mills approach, we replace the constant parameters λ_i in (1) by arbitrary scalar superfunctions $\Lambda_i(x,\theta)$:

$$V'_m(x,\theta) = [\exp(i\Lambda_i(x,\theta)T_i)]_{mn} V_n(x,\theta). \tag{2}$$

These transformations form a group, since the product of scalar superfunctions is again a scalar superfunction.[4,9] By the same token, we attained the most general localization of the group of internal symmetries in superspace.[3)]

3. Just as in the Yang-Mills theory, we can define a lengthened covariant derivative

$$\Delta_\alpha V_m(x,\theta) = (D_\alpha + ig\Psi_{\alpha i}(x,\theta)T_i)_{mn} V_n(x,\theta). \tag{3}$$

We have introduced here the "compensating" superfield $\Psi_\alpha(x,\theta) = \Psi_{\alpha i}(x,\theta)t_i$, where t_i are generators of the adjoint representation of the group. It is easy to verify that $\Delta_\alpha V_m$ is transformed in accord with the same law (2) as the superfield V_m itself, if $\Psi_\alpha(x,\theta)$ is transformed in accord with the law

$$\Psi'_\alpha(x,\theta) = e^{ig\Lambda(x,\theta)}\Psi_\alpha(x,\theta)e^{-ig\Lambda(x,\theta)} - \frac{i}{g}e^{ig\Lambda(x,\theta)}D_\alpha e^{-ig\Lambda(x,\theta)}. \tag{4}$$

Here $\Lambda(x,\theta) = \Lambda_i(x,\theta)t_i$. The group property of (4) is obvious.

4. It is now easy to construct for the superfields V_m a Lagrangian that is invariant to the gauge transformations (2). To this end it is sufficient to lengthen all the spinor derivatives D_α in the initial Lagrangian in accord with the rule (3). By the same token we include the invariant interaction with the gauge superfield.

The next step is to write down for Ψ_α an interaction Lagrangian that is invariant relative to (4). Leaving out the details, we indicate that on the basis of the connection with the projection operators[12] this Lagrangian is fixed in the form

$$L = \frac{1}{16}\,Tr\left\{\bar{\Psi} i \not{\partial}\Psi - \frac{1}{2}[(\bar{D} + ig\bar{\Psi})\gamma_\mu\Psi]^2 + \frac{1}{12}[(\bar{D} + ig\bar{\Psi})\sigma_{\mu\nu}\Psi]^2\right\}, \tag{5}$$

where Tr stands for the trace of the internal-symmetry indices, and the action takes the invariant form $S = \int d^4x d^4\theta L(x,\theta)$.[11,9] We emphasize that the field Ψ_α enters in (5) polynomially.

Notice should be taken of a pecularity, not yet fully explained, which may turn out to be a source of difficulties. In the free case, when $g = 0$, the Lagrangian is invariant not only to the transformation $\Psi'_\alpha = \Psi_\alpha + D_\alpha\Lambda$ which follows from (4), but also to the transformation

$$\Psi'_\alpha(x,\theta) = \Psi_\alpha(x,\theta) + (\not{\partial}\gamma_5 D)_\alpha\Omega(x,\theta), \tag{6}$$

where Ω is an arbitrary scalar superfield. We do not know whether this invariance can be generalized in the case of an interaction with $g \neq 0$.

5. To clarify the meaning of the results, let us discuss them in terms of fields. The superfield $\Psi_\alpha(x,\theta)$ has an expansion

$$\Psi_\alpha(x,\theta) = \psi_\alpha^{(1)}(x) + \bar{\theta}^\beta \phi_{\beta\alpha}^{(1)}(x) + \frac{1}{4}\,\bar{\theta}\theta\,\psi_\alpha^{(2)}(x)$$

$$+ \frac{1}{4}\,\bar{\theta}\gamma_5\theta\psi_\alpha^{(3)}(x) + \frac{1}{4}\,\bar{\theta}\, i\gamma_\mu\gamma_5\theta\psi_\alpha^{\mu}(x) + \frac{1}{4}\,\bar{\theta}\theta\bar{\theta}^\beta\phi_{\beta\alpha}^{(2)}(x) \tag{7}$$

$$+ \frac{1}{32}(\bar{\theta}\theta)^2\psi_\alpha^{(4)}(x).$$

It contains four Majorana spinor fields $\psi_\alpha^{(i)}(x)$, a Majorana spin-vector field $\phi_{\beta\alpha}(x)$, and two real boson fields $\phi_{\beta\alpha}(x)$:

$$\phi_{\beta\alpha}(x) = (v(x)\,1 + a(x)\gamma_5 + iv^\mu(x)\gamma_\mu + iA^\mu(x)\gamma_\mu\gamma_5 + ie^{\mu\nu}(x)\sigma_{\mu\nu})_{\beta\alpha}\,. \tag{8}$$

Calculations in terms of the fields are quite cumbersome. We present the result only for the case of free fields ($g=0$). The gauge invariance (4), and also (6), make a number of degrees of freedom from the expansion (7)—(8) arbitrary (harmless), and they do not appear in the equations of motion. Some other components of the expansion turn out to be equal to zero by virtue of the equations of motion. As a result, in the free case the Lagrangian (5) describes a massless vector field $V_\mu(x)$ and massless spin-vector field $\chi_{\mu\alpha}(x)$, which obey the Proca and Rarita-Schwinger equations

$$\Box V_\mu - \partial_\mu\partial^\nu V_\nu = 0;\ \not{\partial}\chi_\mu - \partial_\mu\gamma^\nu\chi_\nu - \gamma_\mu\partial^\nu\chi_\nu + \gamma_\mu\not{\partial}\gamma^\nu\chi_\nu = 0\,.$$

We note that the invariance transformations of these equations

$$V_\mu \to V_\mu + \partial_\mu b; \qquad \chi_\mu \to \chi_\mu + \partial_\mu\lambda$$

are connected with transformations (4) and (6), respectively.

Details of the calculations and an analysis of the proposed theory with the interaction (5) will be the subject of further publications.

1) These theories have stimulated a large number of studies that made clear their renormalizability,[5-7] the possibility of spontaneous violation, and the construction of models of unified theories of electromagnetic and weak interaction.[7] An analogy between the CGST and the "quasi" Yang-Mills theories[10] was noted in[9].

2) We use the notation $\not{\partial} = \gamma^\mu\partial_\mu$; $(\gamma_\mu)^+ = g_{\mu\mu}\gamma_\mu$, $g_{\mu\nu} = \mathrm{diag}(+---)$, $\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$, and $\sigma_{\mu\nu} = (i/2)[\gamma_\mu,\gamma_\nu]$.

3) In the CGST,[1-4] scalar superfunctions $\Lambda_i(x,\theta)$ of limited type (chiral) are used and, accordingly, chiral material superfields $V_m(x,\theta)$ are considered. In the GGST the material superfields should be of general form.

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