Dynamic reconstruction of symmetry and limitations on the masses and coupling constants in the Higgs model

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It is shown that at definite relations between the coupling constants the radiative corrections lead to the absence of symmetry breaking in the Higgs model.

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The question of the influence of radiative corrections on symmetry breaking in gauge theories has been intensely investigated of late. The first and most significant paper in this direction is that of Coleman and Weinberg.[1] They, however, used renormalization conditions that fixed a number of unphysical parameters at different and furthermore nonequilibrium values of classical fields. This makes the interpretation of the results of[1] extremely difficult[1] and leads to a number of semiterminological misunderstandings. A similar investigation undertaken within the framework of standard renormalization procedure in[2] contained an analysis of only the model of the scalar field $\lambda\phi^4$, and the approximation contained in[3] was insufficient for a study of dynamic effects.

In this paper we use a standard renormalization procedure in the investigation of symmetry breaking in the Higgs model that describes the interaction of a vector field A_μ and a complex scalar field

$$L = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + (\partial_\mu + igA_\mu)\psi^*(\partial_\mu - igA_\mu)\psi + \mu^2\psi^*\psi - \lambda(\psi^*\psi)^2. \qquad (1)$$

After symmetry breaking the field ψ acquires a c-number part

$$\phi_c/\sqrt{2}: \quad \psi \longrightarrow \frac{1}{\sqrt{2}}(\phi + i\chi + \phi_c);$$

where χ is an unphysical Goldstone field.

We consider the effective potential $V(\phi_c)$[1,4] corresponding to the Lagrangian (1) with normalization conditions imposed at the point of the minimum of $V(\phi_c)$ at $\phi_c = \delta \equiv \mu/\sqrt{\lambda}$

$$\left.\frac{\partial V}{\partial \phi_c}\right|_{\phi_c=\sigma} = 0, \qquad \left.\frac{\partial^2 V}{\partial \phi_c^2}\right|_{\phi_c=\sigma} = 2\mu^2. \qquad (2)$$

The physical meaning of the conditions (2) is that the position of the minimum of $V(\phi_c)$ at $\phi_c = \sigma \neq 0$ and of its curvature at this point are fixed in at the values obtained from (1) in the classical approximation.

We consider for simplicity the case $\lambda \ll g^2$. The contribution of the scalar particles in the expression for $V(\phi_c)$ can then be neglected, and standard calculations[1,4,5] lead to the effective potential

$$V(\phi_c) = \frac{\lambda\phi_c^4}{4} - \frac{\mu^2\phi_c^2}{2} + \frac{3g^4}{64\pi^2}\left(\phi_c^4 \ln\frac{\phi_c^2}{\sigma^2} - \frac{3}{2}\phi_c^4 + 2\sigma^2\phi_c^2\right) + V(0). \qquad (3)$$

As follows from[1,5], the higher orders of perturbation theory can modify significantly expression (3) only in the region of asymptotically large $\phi_c \sim \sigma\exp(1/g^2)$. We, however, are now interested in the region $\phi_c \lesssim \sigma$. In this case it follows from (3) that at $\lambda < 3g^4/16\pi^2$ the potential $V(\phi_c)$ acquires a new minimum at $\phi_c = 0$. It is easy to verify that the same effective potential is obtained in the scheme of[1] if the parameters g^2 and λ, which are determined by the conditions (2), are connected with the corresponding parameters of[1] with the aid of the equation of the renormalization group.[1,4] In this sense, our results are equivalent to those of Coleman and Weinberg. The study of the massless electrodynamics

$$\left(\frac{\partial^2 V}{\partial \phi_c^2}\bigg|_{\phi_c=0} = 0\right)$$

and the massive electrodynamics

$$\left(\frac{\partial^2 V}{\partial \phi_c^2}\bigg|_{\phi_c=0} > 0\right)$$

(in the terminology of Coleman and Weinberg) reduces to a study of the usual Higgs model at $\lambda = 3g^4/16\pi^2$ and $\lambda < 3g^4/16\pi^2$, respectively, and the minimum at $\phi_c \neq 0$, which appears dynamically in the scheme of[1], is in our approach the ordinary "classical" minimum that leads to spontaneous symmetry breaking.

The use of a standard renormalization procedure not only makes obvious the physical meaning of the results of[1], but yields also new physical information. Namely, it follows from (3) that at $\lambda < 3g^4/32\pi^2$ the minimum at $\phi_c = 0$ is deeper than the minimum at $\phi_c = \sigma$. Thus, symmetry breaking occurs in the Higgs model only if

$$\lambda > \frac{3g^4}{32\pi^2} \tag{4}$$

or, in the language of scalar and vector fields, at

$$m_\phi^2 > \frac{3g^2}{16\pi^2}\, m_A^2 \;. \tag{5}$$

We recognize that the constant λ defined by the conditions (2) has the meaning of the parameter μ^2/σ^2, but is not equal to the coupling constant λ_{int} of the scalar field ϕ on the mass shell. In the tree approximation we have $\lambda = \lambda_{\text{int}}$, but in the single-loop approximation at $m_\phi \ll m_A$ (i.e., at $\lambda \ll g^2$) we have

$$\lambda_{int} = \frac{1}{6}\,\frac{\partial^4 V}{\partial \phi_c^4}\bigg|_{\phi_c=\sigma}$$

and it follows from (3) that $\lambda_{\text{int}} = \lambda + g^4/2\pi^2$. At the same time, $g_{\text{int}}^2 = g^2 + O(g^4) \approx g^2$.

We note that the constant λ is positive "by definition": $\lambda \equiv \mu^2/\sigma^2$. The potential $V(\phi_c)$ can thus have a minimum at $\phi_c = \sigma \neq 0$ only if

$$\lambda_{int} > \frac{g_{int}^4}{2\pi^2}\,, \tag{6}$$

and this minimum, according to (4), is stable (i.e., deeper than the minimum at $\phi_c = 0$) when

$$\lambda_{int} > \frac{19}{32\pi^2}\, g_{int}^4 \;. \tag{7}$$

For a numerical estimate we take $g^2/4\pi \sim 10^{-2}$ and $m_A \sim 10^2$ GeV, just as in Weinberg's model. [6] It follows then from (6) and (7) that

$$\lambda_{int} > 10^{-3}\;,$$

and from (5) it follows that

$$m_\phi > 5 \text{ GeV}.$$

On the other hand, if we consider the theory of strong interactions ($g \gtrsim 1$, $m_A \sim 1$ GeV), then we obtain respectively $\lambda > 10^{-1}$ and $m_\phi > 100$ MeV.

We note that in theories with $V(\sigma) \sim V(0)$ a first-order phase transition with reconstruction of the symmetry[7] will take place at a rather low temperature T_c($T_c \to 0$ as $V(\sigma)) \to V(0)$, i.e., in our case as $\lambda_{\text{int}} \to 19 g_{\text{int}}^4/32\pi^2$). This makes the models with $V(\sigma) \sim V(0)$ nonrelativistic, see, e.g., [8]. The absence of a phase transition in stars makes it possible to strengthen the restrictions (4)–(7). The results of the present paper can be trivially generalized to include also nonabelian theories. We note that the limitations obtained in this manner on the constant λ_{int} and on the Higgs-meson mass m_ϕ are much stronger than the existing experimental restrictions.

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