Contribution of two-photon mechanism to the real part of the $K_L \to \mu\bar{\mu}$ decay amplitude and estimates of the masses of charmed particles

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It is shown that allowance for the contribution of the two-photon state to the real part of the $K_L \to 2\mu$ decay amplitude can alter substantially the estimates of the masses of charmed quarks.

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In the Weinberg-Salam unified theory of weak and electromagnetic interactions,[1] supplemented by the Glashow-Iliopoulos-Maiani quark scheme,[2] the amplitude of the transition $(\bar{n}\lambda)(l^+l^-)$ depends on the mass of the charmed quark, so that the value of m_c can be estimated from data on $K_L \to 2\mu$ decays. In[3–7] the estimates of m_c were based on a comparison of the experimental data with theoretical value of the real part of the $K \to 2\mu$ amplitude, in the calculation of which only the contributions of intermediate bosons were taken into account. Yet the contribution of the two-photon state to the real part is of the same order of magnitude as the contribution of the intermediate bosons, so that allowance for this contribution can be important. We present in this note the results of calculations of the two-photon contribution to the real part of the amplitude of the $(\bar{n}\lambda)(l^+l^-)$ transition. We used in the calculations Feynman's gauge, in which the W-boson propagator is of the form $i\delta_{\mu\nu}/(K^2 - M^2)$. In this gauge, from among the numerous diagrams responsible for the $(\bar{n}\lambda) \to 2\gamma \to (l^+l^-)$ transition, a contribution on the order of $G\alpha^2$ is made only by the diagrams of Fig. 1. The results obtained under the assumption $m_p \ll m_c \ll m_W$, for the real part of the sum of the three diagrams, is

$$\mathrm{Re}M^{(2\gamma)} = \frac{G\alpha^2 \sin\theta\cos\theta}{4\sqrt{2}\pi^2}\left\{3\left(\frac{e_p^2}{e^2}\right)\ln\frac{m_c^2}{m_p^2} - \left[\frac{e_p e_n}{e^2}\left(\ln\frac{m_c^2}{m_p^2} + \frac{10}{3}\right)\ln\frac{m_c^2}{m_p^2}\right]\right\}(\bar{u}_n\gamma_\mu(1+\gamma_5)u_\lambda)(\bar{u}_l\gamma_\mu\gamma_5 u_l); \tag{1}$$

where m_c and m_p are the masses of the charmed and ordinary p-quark, respectively, while e_p, e_n, and e are the charges of the p-quark, the n-quark, and the electron; θ is the Cabibbo angle. The term in the square bracket of (1) is due to diagrams b and c of Fig. 1.

The real part of the amplitude of the transition $(\bar{n}\lambda)$ into the $^1S_0$ state of the l^+l^- pair, due to the intermediate W and Z bosons, is[3]

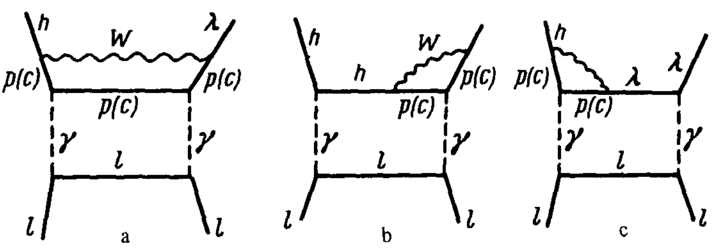

FIG. 1.

$$M^{(W,Z)} = -\frac{G^2 m_c^2 \sin\theta\cos\theta}{4\pi^2}(\bar{u}_n\gamma_\mu(1+\gamma_5)u_\lambda)(\bar{u}_l\gamma_\mu\gamma_5 u_l). \tag{2}$$

Therefore the real part of the quark transition of $(\bar{n}\lambda)$ to the ${}^1S_0$ state of (l^+l^-) as a whole is given by the formula

$$\mathrm{Re}\,M = -\frac{G\sin\theta\cos\theta}{4\pi^2}\left[Gm_c^2 - \frac{3a^2}{\sqrt{2}}\left(\frac{e_p^2}{e^2}\right)\ln\frac{m_c^2}{m_p^2} + \frac{a^2}{\sqrt{2}}\frac{e_pe_n}{e^2}\left(\ln\frac{m_c^2}{m_p^2} + \frac{10}{3}\right)\ln\frac{m_c^2}{m_p^2}\right]\left(\bar{u}_n\gamma_\mu(1+\gamma_5)u_\lambda\right)(\bar{u}_l\gamma_\mu\gamma_5u_l). \tag{3}$$

The amplitude (3) was calculated neglecting the strong interactions of the quarks. The corrections that arise when the latter are taken into account can be estimated within the framework of the asymptotically free gauge theory in which the strong interaction of the colored quarks is realized by an octet of colored gluons. If we assume in addition that the gluons are electrically neutral (this is so if $e_p^2/e^2 = 4/9$ and $e_pe_n/e^2 = -2/9$), then the principal corrections to $M^{(2\gamma)}$ are due to renormalization of the weak amplitude of the process $\bar{n}\lambda \to \bar{p}p$ $(\bar{c}c)$ by the strong interactions, and are expressed in terms of the coefficients[8] c_+ and c_- in the Wilson expansion of the T-product[9] of the operators of the hadronic weak currents at short distances. Following[8], we write down the effective Lagrangian for the process $\bar{n}\lambda \to \bar{p}p$ $(\bar{c}c)$ in the form

$$L_W = -\frac{G}{\sqrt{2}}\sin\theta\cos\theta : \left\{\frac{c_-}{2}[(\bar{n}p)(\bar{p}\lambda) - (\bar{n}\lambda)(\bar{p}p)] + \frac{c_+}{2}[(\bar{n}p)(\bar{p}\lambda) + (\bar{n}\lambda)(\bar{p}p)] - (p \leftrightarrow c)\right\} : + \text{h.c.} \tag{4}$$

with coefficients c_+ and c_-, which depend on a certain characteristic momentum p^2 in accordance with the law[8]

$$c_-(p^2) = (c_+(p^2))^{-2} = \left[1 + \frac{25}{3}\frac{g^2(p^2)}{16\pi^2}\ln\left(\frac{M_W^2}{p^2}\right)\right]^{12/25}, \tag{5}$$

where $g(p^2)$ is the effective charge of the quark-gluon interaction, which also depends on p^2 in accordance with the formula of Gross and Wilczek[10]

$$\frac{g^2(p_1^2)}{g^2(p_o^2)} = \left[1 + 9\frac{g^2(p_o^2)}{16\pi^2}\ln\left(\frac{p_1^2}{p_o^2}\right)\right]^{-1}, \tag{6}$$

in which it is implied that $p_0^2 \ll p_1^2 \lesssim 4m_c^2$, so that the c-quarks are not yet excited.

The function Re$M^{(2\gamma)}$ modified by the strong interaction is approximately described by the formula

$$\mathrm{Re}\,M^{(2\gamma)} = \frac{Ga^2}{3\sqrt{2}\,\pi^2}\sin\theta\cos\theta\left\{c(4m_c^2)\ln\frac{4m_c^2}{m_k^2} - c(4m_p^2)\ln\frac{4m_p^2}{m_k^2}\right. \tag{7}$$

$$\left. + \frac{1}{3}\left(\frac{5}{3} - 2\ln 2\right)c(4m_c^2)\ln\frac{m_c^2}{m_p^2} + \frac{1}{3}J\right\}(\bar{u}_n\gamma_\mu(1+\gamma_5)u_\lambda)(\bar{u}_l\gamma_\mu\gamma_5 u_l),$$

where

$$c = 2c_+ - c_-, \text{ a } \quad J = \int_{4m_p^2}^{4m_c^2} c(s)\sqrt{1 - 4m_p^2/s}\,\ln(s/m_p^2)\,ds/s.$$

Numerically, the effect of the strong interactions leads to a decrease of the amplitude (1) by approximately a factor 1.5, but the accuracy of the estimate of this factor is low. Figure 2 shows Re$M^{(2\gamma)}$ (curve a—without allowance for

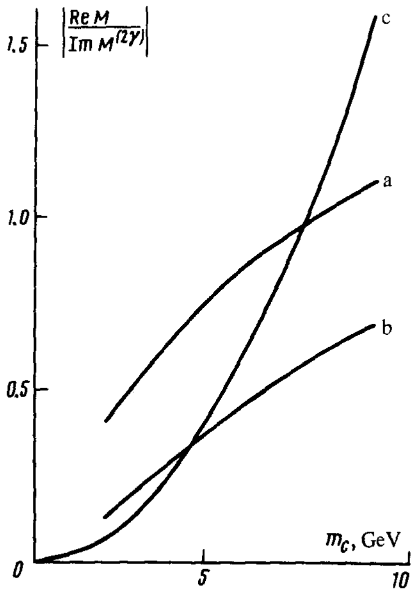

FIG. 2.

strong interactions, curve b—with) and $M^{(W,Z)}$ (curve c), normalized to the quantity $|\mathrm{Im}M^{(2\gamma)}|$ obtained by using the experimental value of the $K_L \to 2\gamma$ vertex. With respect to curve b, it must be borne in mind that its position can change somewhat in more correct calculations. Curve c corresponds to $-M^{(W,Z)}$ without allowance for the strong interactions, since there are discrepancies in the published estimates of the gluon corrections to $M^{(W,Z)}$ (see[6,7]). All the diagrams were constructed under the assumption that $m_p \approx 0.5$ GeV, $M_W \approx 100$ GeV, and $g^2(1\ \mathrm{GeV}^2)/4\pi \approx 1$.

It is seen from Fig. 2 that at $m_c \approx 3$—7 GeV a strong mutual cancellation of $\mathrm{Re}M^{(2\gamma)}$ and $M^{(W,Z)}$ can occur, so that the determination of m_c from a comparison of the calculated and experimentally measured value of ReM in the $K_L \to 2\mu$ decay may turn out to be an ambiguous procedure. The impression that the $K_L \to 2\mu$ decay is not very effective for estimates of m_c better than at $m_c \lesssim 10$ GeV becomes even stronger if one more circumstance is taken into account. Namely, in estimates of m_c one uses quark diagrams that are significant only at short distances ($\lesssim (1\ \mathrm{GeV})^{-1}$). If we put in formulas (3) and (4) of[4] $m_p = 0.5$ GeV, which corresponds precisely to allowance for distances $\lesssim 1/2m_p \approx (1\ \mathrm{GeV})^{-1}$, then the contribution of the quark diagrams to the $K_L \to 2\gamma$ vertex, and hence to $\mathrm{Im}M^{(2\gamma)}$, turns out to be small ($\sim 3\%$) in comparison with the experimental value. It is therefore possible that the real part is also determined to a considerable degree by large distances ($\sim m_k^{-1}$). For a rough estimate of the contribution of large distances to $\mathrm{Re}M^{(2\gamma)}$ we can use the relation

$$\mathrm{Re}\, M^{(2\gamma)}/\mathrm{Im} M^{(2\gamma)} \sim \frac{3}{\pi}\left(\ln \frac{m_k^2}{m_\mu^2}\right)^{-1} \sim 1/3.$$

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