On the properties of liquid He^3

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It is shown that liquid He^3 is close to an antiferromagnetic transition. The dependence of the heat capacity on the temperature is determined. The relations between the parameters of the Landau Fermi-liquid theory are obtained.

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1. Liquid He^3 is well described by the Landau theory only at $T<0.1°$. The parameters of this theory are anomalously large. When the pressure is changed from 0 to 27 atm, the quantities Φ_0 and Φ_1,[1] which are connected with the values of the speed of sound and of the effective mass, increase from 10 and 6 to 100 and 15. No such large quantities can appear in a theory where there is no small parameter. The large value of m^* means that an interaction exists between the quasiparticles of the He^3 and depends strongly on their velocity and energy; this interaction is connected with exchange of particle-hole excitations with a large statistical weight. It is shown in this paper that such excitations can be virtual paramagnons with wave vectors $k_0 \neq 0$, meaning that He^3 is close to an antiferromagnetic transition. In other words, we shall assume that the magnetic susceptibility of He^3 as a function of the wave vector has a sharp maximum at $k=k_0$, where k_0 is of atomic order.

2. In terms of the scattering amplitude Γ, this means that the static Γ is of the form

$$\frac{a^2 p_F^2}{\pi^2 v}\Gamma(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}) = \frac{3}{2} D(\mathbf{p}_1 - \mathbf{p}_2) - \vec{\sigma}_1\vec{\sigma}_2\left\{\frac{1}{2} D(\mathbf{p}_1 - \mathbf{p}_2) + D(\mathbf{k})\right\}. \quad (1)$$

D has the meaning of the paramagnon propagation function, and we parametrize its k-dependence at $k \approx k_0$

$$D^{-1}(k) = \xi^2 + \gamma^2\left[\left(\frac{k^2}{k_0^2}\right) - 1\right]^2 \quad (2)$$

the parameter ξ determines the proximity of the liquid to the phase transition, $\xi^2 \ll 1$, and the quantity γ^2 is connected with the dispersion of the magnetic susceptibility $\chi(k)$ at $k \approx k_0$: $\chi(k) \sim D(k)$. Expression (1) was obtained by separating the resonance D in the direct and exchange channels of Γ. The scalar part of Γ depends essentially on the angle between $\mathbf{p}_1$ and $\mathbf{p}_2$; the spin part has a maximum with respect to the momentum transfer k at $k=k_0$. In the limit as $k \to 0$ and $p_1=p_2=p_F$, the amplitude Γ depends only on the angle between $\mathbf{p}_1$ and $\mathbf{p}_2$, and can be connected with the parameters of the Landau theory[1,2]

$$\frac{a^2 p_F^2}{\pi^2 v}\Gamma(k=0) = A(x) - \vec{\sigma}_1\vec{\sigma}_2\left\{\frac{1}{3} A(x) + D(k=0)\right\}. \quad (3)$$

The function $A(x)$ is given by

$$A(x) = \frac{2}{\pi} A_0 \frac{\kappa}{\kappa^2 + (x - x_0)^2}, \quad x = \frac{\mathbf{p}_1 \mathbf{p}_2}{p_F^2}, \quad x_0 = 1 - \frac{k_0^2}{2 p_F^2}. \tag{4}$$

The quantities κ and A_0 are expressed in terms of ξ, γ, and k_0

$$\kappa = \frac{\xi k_0^2}{2 p_F^2 \gamma} \qquad A_0 = \frac{3}{8} \frac{k_0^2}{p_F^2} \frac{\pi}{\xi \gamma} \tag{5}$$

Calculating the first harmonics of A with respect to x and comparing them with the Landau theory, we obtain the parameters $A_0 = 0.9$ and $x_0 = 0.075$. From the data on the heat capacity and (5) we can determine also ξ^2, γ^2, and κ:

$$\xi^2 = 0.06, \quad \gamma^2 \cong 5, \quad \kappa = 0.025, \quad k_0^2 = 0.5\, p_F^2 .$$

Thus, the angular dependence of A is characterized by a narrow peak at $x = x_0$. Expression (4) enables us to find the following harmonics of A, meaning also the parameters Φ_i and Z_i of the Landau theory[1,2]:

$$\Phi_2 = 3, \quad \Phi_3 = 0.5, \; \Phi_4 = -1.5, \; Z_1 = -0.7, \; Z_2 = -0.5, \; Z_3 = -0.1,$$

$$Z_4 = 0.6.$$

The large value of Φ_2 allows us to conclude that transverse zero sound ($m = 1$) can propagate in He^3. It is known that the theory in which the two harmonics Φ_0 and Φ_1 are taken into account leads to the requirement $\Phi_1 > 6$; at $\Phi_1 < 6$ no zero sound can propagate. The true value $\Phi_1 = 6.25$ is very close to the threshold $\Phi_1 = 6$.

3. We determine the connection between the parameters ξ, γ, and k_0, on the one hand, and the elementary-excitation spectrum, on the other. This can be done by separating the contribution of Γ to the main mass Σ of the particles. The proper mass Σ is an analytic function of p^2 as $\xi^2 \to 0$, but has a singularity in ϵ as $\xi^2 \to 0$, so that Σ can be expanded in powers of $p^2 - p_F^2$, and the dependence on ϵ must be taken into account exactly. Calculation leads to the following dependence of the Green's function G of the particles on p^2 and ϵ:

$$G^{-1}(p^2, \epsilon) = \epsilon - \frac{p^2 - p_F^2}{2m_0^*} + \epsilon \frac{1 - a}{a} \frac{1}{\sqrt{1 - i\frac{|\epsilon|}{\epsilon_0}} + 1}. \tag{6}$$

The jump in the Fermi occupation a of the particles is very small and is expressed in terms of the parameter Φ_0 of the Landau theory, namely $a = (1 + \Phi_0)^{-1} = 0.08$. The quantity m_0^* is defined by the relation

$$\frac{m}{m_0^*} = 1 + \left.\frac{\partial \Sigma}{\partial p^2}\right|_{p_F^2} \times 2m$$

and is connected with $m_1^* a$ and with the speed of sound c

$$m_0^* = a m^*, \qquad c^2 = p_F^2 / 3 m m_0^*, \qquad m_0^* \cong 0.25\, m \tag{7}$$

The energy ϵ_0 in (6), which is characteristic of G, determines the end point E_p of the quasiparticle spectrum

$$\epsilon_0 = \frac{2k_0 v}{\pi}\xi^2 = 0.09^\circ, \quad E_p = \epsilon_p - i\gamma_p = v(p-p_F) - \frac{i}{4}\frac{\epsilon_p|\epsilon_p|}{\epsilon_0}.$$

Well-defined quasiparticles exist at $\epsilon < 0.1^\circ$; at $\epsilon < 0.1^\circ$ the damping γ_p becomes comparable with ϵ_p. At $\epsilon > 0.1^\circ$ the pole of G goes off to the "unphysical" sheet of the ϵ plane:

$$\epsilon_p = -i\frac{|p-p_F|(p-p_F)}{2m_1}, \quad m_1 \approx 0.5\,m.$$

Thus, at $\epsilon > \epsilon_0$ the pole of G corresponds to diffuse one-particle excitation. We shall show below that this leads to a dependence of the heat capacity on T in the form $C \sim \sqrt{T}$.

4. That part of the free energy of He^3 which depends essentially on ξ^2 and T can be obtained by summing the "dangerous" ring diagrams in which $D(k)$ is the simplest element. A similar calculation for an almost ferromagnetic liquid was carried out in[3]:

$$F = F_0 + \frac{3}{2}\int d^3kT\sum_{\omega_n}\ln\left(1 - B_0(k)\frac{\pi|\omega_n|}{2kv}\right) + \frac{1}{2}\int d^3kT\sum_{\omega_n}\ln\left(1 - A_0\frac{\pi|\omega_n|}{2kv}\right). \tag{8}$$

The first term in (8) is connected with the spin-density fluctuations and contains the zeroth harmonic of the spin part Γ: $B_0(k) = -(1/3)(1-a) - D(k)$. The second term in (8) is connected with fluctuations that do not affect the spin and contains the zeroth harmonic of the scalar part of Γ: $A_0 = 1 - a \approx 1$. The calculation leads to the following dependence of the heat capacity C on T:

$$C = C_0(T)\left\{a + \frac{1-a}{\left(1+\frac{T^2}{T_2^2}\right)^{1/2}}\left[\frac{2}{1+\frac{T^2}{T_1^2}+\left[\frac{T^2}{T_0^2}+\left(1+\frac{T^2}{T_1^2}\right)\right]^{1/2}}\right]^{1/2}\right\},$$

$$T_0 = \frac{3}{\pi^2}\epsilon_0, \quad T_1 = \frac{T_0}{\xi}\sqrt{\frac{3}{A_0}}, \quad T_2 = \frac{9}{\pi^2}\frac{\epsilon_0}{\xi^2}\frac{1}{A_0}, \quad \epsilon_0 = \frac{2k_0 v}{\pi}\xi^2 \tag{9}$$

$C_0(T)$ denotes the limit of C as $T \to 0$: $C_0(T) = \frac{1}{3}p_F m^* T$.

At small T, Eq. (9) takes the simpler form

$$C = C_0(T)\left\{a + (1-a)\left[\frac{2}{1+\left(1+\frac{T^2}{T_0^2}\right)^{1/2}}\right]^{1/2}\right\}. \tag{10}$$

Figure 1 shows the plot of C against $\sqrt{T}$ corresponding to formula (10). It is seen that C is a linear function of $\sqrt{T}$ at $T > 0.03^\circ$. The curve on Fig. 1 corresponds to the values $\xi^2 = 0.06$, $T_0 = 0.03^\circ$, $T_1 = 0.2^\circ$, and $T_2 = 1.5^\circ$. After determining ξ^2 in the region of small T from comparison with experiment, we can obtain the dependence of C on T for large T from (9). This dependence corresponds to curve 1 on Fig. 2. At $T > 0.5^\circ$, the function $C(T)$ is linear in T

$$\frac{\partial C}{\partial T} = \frac{1}{3} p_F m_0^* , \quad m_0^* = a m^* . \tag{11}$$

Relation (11) reveals the physical meaning of the effective mass m_0^* defined in (7) and establishes the connection between the speed of sound c and $C(T)$ at large T

$$\partial C(T)/\partial T = p_F^2/9mc^2 .$$

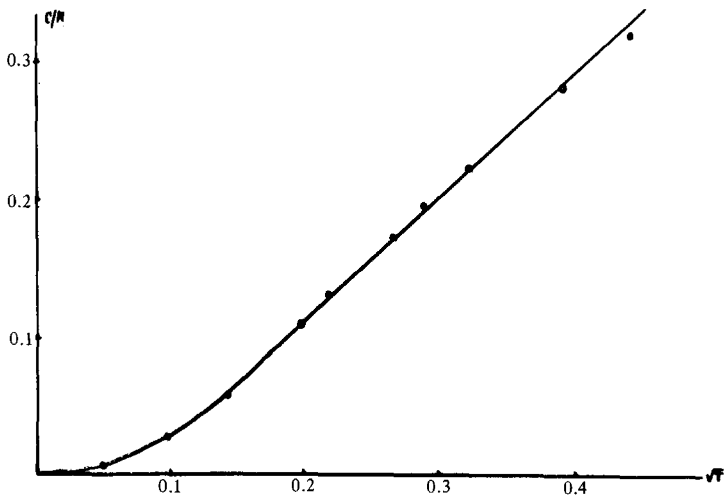

FIG. 1. Dependence of C on $\sqrt{T}$ at $T < 0.2°$.

Curve 1 on Fig. 2 corresponds to 2% accuracy of the theory at $T < 0.3°$ and to 20% at $T > 0.3°$. Curve 2 corresponds to the value $\xi^2 = 0.035$ and describes more accurately the course of the heat capacity at large T. The true value of ξ^2 lies between 0.035 and 0.06. These values lead to an estimate of the maximum value of the magnetic susceptibility $\chi(k_0)$

$$50 < \frac{\chi(k_0)}{\chi_0(0)} < 85, \qquad \frac{\chi(0)}{\chi_0(0)} \approx 9.$$

5. Thus, the theory developed above permits a quantitative description of the properties of He^3 in a wide interval of T. The temperature $T = 0.5° \ll \epsilon_F$ is the analog of ω_D for a solid and has the meaning of the paramagnon-gas degeneracy temperature. The Fermi-liquid theory is applicable both at low temperatures $T < 0.05°$ and at high ones $T > 0.5°$.

Let us list briefly other arguments favoring the proximity of He^3 to an antiferromagnetic transition; these arguments will be considered in detail in subsequent communications:

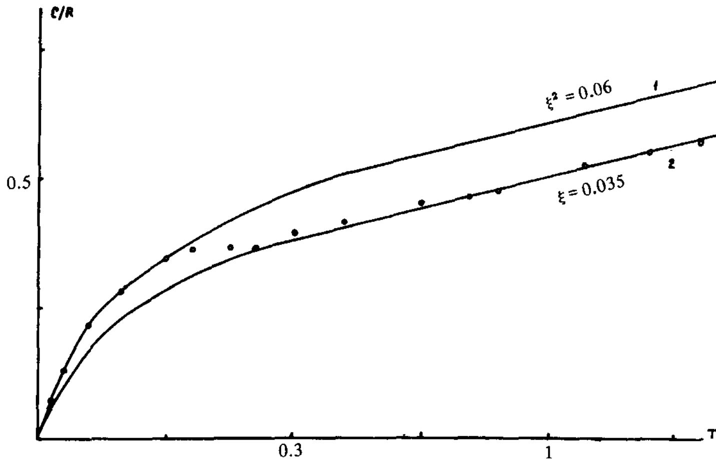

FIG. 2. Dependence of C on T at $T < 1.5°$ for two values of ξ^2.

1) The paramagnons decrease the phase volume of the quasiparticles at the Fermi surface, and this draws out the transition to the superfluid state into the region of small T.

2) The strong dependence of Γ on the angle between $\mathbf{p}_1$ and $\mathbf{p}_2$ contributes to pairing with nonzero angular momentum.

3) The viscosity of an almost-antiferromagnetic liquid is of the order of $1/\sqrt{T}$, and the thermal conductivity is constant ($0.05° < T < 0.5°$). A viscosity dependence proportional to $1/\sqrt{T}$ was observed in experiment.[4]

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[1]L. D. Landau, Zh. Eksp. Teor. Fiz. **30**, 1058 (1956) [Sov. Phys.-JETP **3**, 920 (1956)].

[2]L. D. Landau, Zh. Eksp. Teor. Fiz. **32**, 59 (1957) [Sov. Phys.-JETP **5**, 101 (1957)].

[3]A. I. Larkin and V. I. Mel'nikov, Pis'ma Zh. Eksp. Teor. Fiz. **20**, 386 (1974) [JETP Lett. **20**, 143 (1974)].

[4]K. N. Zinov'eva, Zh. Eksp. Teor. Fiz. **34**, 609 (1958) [Sov. Phys.-JETP **7**, 421 (1958)].