Kinetic transition in a system of parametrically excited spin waves

A. S. Mikhaĭlov

Moscow State University
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A kinetic transition accompanied by establishment of long-range order is possible in a system of parametrically excited spin waves (PESW) in the case of parallel pumping in ferro- and antiferromagnets when the temperature is lowered or when the excess above threshold is increased.

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The interaction of spin waves (SW) with a parallel-pumping field is described by the Hamiltonian

$$H_0 = \sum_k \hbar \bar{\omega}_k a_k^* a_k + \frac{\hbar}{2} \sum_k (h V_k^* a_k a_{-k} + \text{c.c.}), \tag{1}$$

in which h is the amplitude of the magnetic pumping field, V_k is a coefficient characterizing the strength of the coupling of the SW with the pump (see[1]). The Hamiltonian (1) is expressed in a "rotating" coordinate system $[a_k \to \exp(i\omega_p t/2) a_k]$, $\bar{\omega}_k \equiv \omega_k - \omega_p/2$, where ω_k and ω_p are the frequencies of the SW and of the external pump. In the considered classical limit, a_k^* and a_k have the meaning of c-number complex amplitudes of the SW.

The Hamiltonian H_0 is reduced to the standard form[2]

$$H_0 = -\sum_{k \in \Omega} s_k p_k q_k + \sum_{k \in \bar{\Omega}} \epsilon_k b_k^* b_k . \tag{2}$$

The decay region of the wave-vector space Ω is defined by the condition

$$|\bar{\omega}_k| < h |V_k| . \tag{3}$$

The remaining vectors **k** belong to the nondecay region $\bar{\Omega}$. The variable p_k and q_k are cononical: p_k are the generalized momenta and q_k are the coordinates; $p_{-k} = p_k^*$ and $q_{-k} = q_k^*$. The complex amplitudes of the SW in the decay region are linearly connected with the variables p_k and q_k:

$$a_k = u_k^{1/2} q_k - \frac{1}{\hbar} (u_k^*)^{1/2} p_k^* ,$$

$$u_k = i (V_k / |V_k|) [(s_k + i\bar{\omega}_k)/2 s_k)], \qquad s_k = \sqrt{h^2 |V_k|^2 - \bar{\omega}_k^2} . \tag{4}$$

In the nondecay region $\bar{\Omega}$, the presence of an external pump leads to a renormalization of the complex amplitudes of the SW ($a_k \to b_k$, the b_k being connected with the a_k by the u, v transformation), and the SW frequency is likewise renormalized in this case:

$$\hbar \bar{\omega}_k \to \epsilon_k , \quad \epsilon_k = \hbar \bar{\omega}_k [1 - (h |V_k| / \bar{\omega}_k)^2]^{1/2} . \tag{5}$$

In experiments on parallel pumping we have $h|V_k| \sim \gamma$ (γ is the SW damping), so that the decay region Ω is small in comparison with the nondecay region $\bar{\Omega}$ [see (3)], and the renormalization (5) is significant in $\bar{\Omega}$ only at the points **k** that are directly adjacent to the decay region Ω. The SW in the nondecay region can be regarded as being in thermal equilibrium, and the entire aggregate of these SW, as well as the system of nonmagnon excitations of the crystal, can be regarded as a thermostat.

The interaction between the PESW and the thermostat will be taken into account by adding to the equations of motion for a_k, in the decay region, the damping $(-\gamma_k a_k)$ and the Gaussian random force.[3] The following Langevin equations are valid for the variables p_k and q_k

$$\dot{p}_k = -(\gamma_k - s_k)p_k + \phi_k(t), \quad \dot{q}_k = -(\gamma_k + s_k)q_k + \psi_k(t) \tag{6}$$

with the random forces ϕ_k and ψ_k.

Since s_k increases with increasing h [see (4)], at a definite value of the pump field amplitude ($h = h_c$) the coefficient $\nu_k = \gamma_k - s_k$ vanishes at certain points **k** ($\mathbf{k} \in \Omega$); this value $h = h_c$ determines the threshold of the parametric resonance. At $h > h_c$, the coefficient ν_k becomes negative and the unstable modes p_k grow exponentially. We note that the coefficient $\mu_k = \gamma_k + s_k$ for the modes q_k does not reverse sign on going through the resonance threshold.

The limitation on the growth of the unstable modes p_k is the result of the nonlinear interaction between the SW. Following Zakharov, L'vov, and Starobinets,[1] we choose as H_{int} the four-magnon reduced Hamiltonian

$$H_{int} = \sum_{kk'} \hbar \left(T a_k^* a_k a_{k'}^* a_{k'} + \frac{1}{2} S a_k^* a_{-k}^* a_{k'} a_{-k'}\right), \tag{7}$$

and for simplicity assume that the coefficients T and S are independent of k and k'. We also assume below that V_k and γ_k do not depend on k, and ω depends only on the modulus of the wave vector k.

By starting from the Hamiltonian (7), we obtain nonlinear equations of motion for the modes p_k—the analog of the Ginzburg—Landau equations with fluctuating force:

$$\begin{cases} \dot{p}_k = -\tilde{\nu}_k(p)p_k + \phi_k(t), \\ \tilde{\nu}_k = \nu_k + \sum\limits_{k_1} \frac{1}{\gamma\hbar^3}(2T\bar{\omega}_k + S\bar{\omega}_{k_1})|p_{k_1}|^2 + \sum\limits_{k_1 k_2} \frac{(2T+S)^2}{2\gamma\hbar^6}|p_{k_1}|^2 |p_{k_2}|^2. \end{cases} \tag{8}$$

ν_k reverses sign on going through the threshold. The coefficient of $|p_{k_1}|^2$ in the expression for $\tilde{\nu}_k$ can be both positive and negative. In view of this it is necessary to take into account the next-higher-order term (the last term in the expression for $\tilde{\nu}_k$).

By constructing in standard fashion[4] from the Langevin equation (8) the corresponding Fokker—Planck equation for the distribution function $\Phi(\{p_k^*, p_k\})$ and solving it in the self-consistent-field approximation,[3] we arrive at the following results:

1. At low excess above threshold, when $\kappa \equiv (h - h_c)/h_c \ll n_0\xi_1$, the stationary regime previously investigated in[5,3] exists; in particular, the correlators $\bar{n}_k = \langle a_k^* a_k \rangle$ are given by

$$\bar{n}_k = (\theta/\epsilon_k^{(1)}), \qquad \epsilon_k^{(1)} = (\hbar\omega_p/2)\left[\frac{(\bar{\omega}_k - \bar{\omega}_o)^2}{2\gamma^2} + \Delta_1^2\right] \quad ,$$

$$\Delta_1^2 = -\frac{1}{2}\kappa + \left[\frac{1}{4}\kappa^2 + (\bar{n}_o\xi_1)^2\right]^{1/2}, \quad \bar{n}_o \equiv (2\theta/\hbar\omega_p) \ , \tag{9}$$

$$\xi_1 \equiv \frac{k_o^2|S|}{2\pi v_o}, \quad v_o \equiv \frac{\partial\omega}{\partial k}\Big|_{k_o}, \quad \bar{\omega}_{k_o} = \bar{\omega}_o = -2T\Sigma\bar{n}_k .$$

Expressions (9) are valid in the region $|\omega_k - \omega_0| \ll \gamma$, and θ is the temperature of the thermostat; for ferromagnets we have $\xi_1 \sim (ak_0)$, where a is the lattice constant.

2. At high excesses above threshold, when $1 > \kappa \gg \bar{n}_0\xi_1$, the solution is essentially different. The corresponding regime can be described as a superposition of a "condensate" filling a sphere of zero thickness in k-space, and "above-condensate" particles, whose distribution is nonsingular. The total distribution takes the form

$$\bar{n}_k = \frac{1}{4\pi k_o^2} N_o \delta(|\mathbf{k}| - k_o) + \bar{n}_k', \quad \bar{n}_k' = (\theta/\epsilon_k^{(2)}),$$

$$\epsilon_k^{(2)} = (\hbar\omega_p/2)\left[\frac{(\bar{\omega}_k - \bar{\omega}_o)^2}{2\gamma^2} + \Delta_2^2\right] , \tag{10}$$

$$\Delta_2 = (\bar{n}_o\xi_2)^{1/2}(2\kappa/\bar{n}_o\xi_2)^{1/6}, \quad \xi_2 \equiv \sqrt{2}\left(\frac{2T+S}{S}\right)\xi_1,$$

$$N_o = \frac{\gamma}{|S|}(2\kappa)^{1/2}, \quad \bar{\omega}_o = \bar{\omega}(k_o) = -2TN_o \ .$$

The "condensate" corresponds to the "single-particle turbulence" analyzed by L'vov[6] using the Wild diagram technique.

3. We present an expression for the absorbed power in the regime (10):

$$Q^{(2)} = (\hbar\omega_p/2)(\gamma^2/|S|)(\bar{n}_o\xi_1)^{1/2}\left[(2\kappa/\bar{n}_o\xi_1)^{1/2} + \left(\frac{2S}{2T+S}\right)^{1/3}(\kappa/\bar{n}_o\xi_1)^{-1/6}\right] \tag{11}$$

4. The asymptotic form of the correlator $g(\mathbf{r}-\mathbf{r}') = \langle n(\mathbf{r})n(\mathbf{r}')\rangle - \langle n\rangle^2$ as $|\mathbf{r}-\mathbf{r}'| \to \infty$ makes it possible to determine the correlation radius r_c of the density fluctuations.[7] For the solution (9), r_c increases monotonically with increasing κ, namely $r_c^{(1)} = (v_0/\gamma)\Delta_1^{-1}$ and $r_c^{(1)} = (v_0/\gamma)(\bar{n}_0\xi_1)^{-1/2}$ at $h = h_c$. In the regime (10), the presence of a "condensate" is reflected in the presence of long-range order in the density n. The correlation radius due to the "above-condensate" particles is

$$r_c = r_{co}\left[\frac{S}{2(2T+S)}\right]^{1/3}\left(\frac{\kappa}{\bar{n}_o\xi_1}\right)^{-1/6}, \tag{12}$$

and decrease with increasing excess above threshold.

5. When h increases in the region of excess above threshold $\kappa \sim \bar{n}_0\xi_1$, a kinetic transition takes place from the stationary regime (9) to the regime (10), and is accompanied by precipitation of the "condensate" and establishment of long-range order. We note that the solution (10) was obtained only in the region $\kappa \gg \bar{n}_0\xi_1$, so that an analysis of the character of the kinetic transition is outside the scope of this paper.

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[1] V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Usp. Fiz. Nauk **114**, 609 (1974) [Sov. Phys. Usp. **17**, 896 (1975)].
[2] V. I. Arnol'd, Matematicheskie metody klassicheskoĭ mekhaniki (Mathematical Methods of Classical Mechanics), Nauka, 1974.
[3] A. S. Mikhaĭlov, Zh. Eksp. Teor. Fiz. **69**, 523 (1975) [Sov. Phys.-JETP **42**, 267(1975)].
[4] H. Haken, Rev. Mod. Phys. **47**, 67 (1975).
[5] V. E. Zakharov and V. S. L'vov, Zh. Eksp. Teor. Fiz. **60**, 2066 (1971) [Sov. Phys.-JETP **33**, 1113 (1971)].
[6] V. S. L'vov, Preprint 7-73, Nuc. Phys. Inst. Siber. Div. USSR Acad. Sci., Novosibirsk, 1973.
[7] A. E. Patashinskiĭ and V. L. Pokrovskiĭ, Fluktuatsionnaya teoriya fazovykh perekhodov (Fluctuation Theory of Phase Transitions), Nauka, 1975.