One possibility of explaining the properties of the ψ meson in *SU*(3) symmetry

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By comparing the probabilities of the decays $\psi\to 3\pi$, $\psi\to K\bar{K}\pi$, and $\psi\to K\bar{K}\eta$ it is possible to identify the representation of the $SU(3)$ symmetry group to which the ψ meson belongs, and to determine whether this meson is a unitary singlet or the eighth component of a unitary octet.

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Since the isotopic spin of the ψ meson is $T=0$, it follows that in $SU(3)$ symmetry, if higher multiplets are not considered, the ψ meson can be either a unitary singlet or the eighth component of a unitary octet (analogous to the η meson in the pseudoscalar octet), or else a mixture of the two. It is possible to ascertain which of the possibilities does take place by investigating the probabilities of the decays of ψ into various hardronic states.[1] In the theory with charmed particles, in which ψ is a bound state of a charmed quark and a charmed antiquark, if the decay of the ψ into hadrons proceeds via virtual gluons, one should expect the hadrons to be produced in this decay in a unitary singlet state. The presence of an octet state of the produced hadron would mean in this case that the ψ decays on account of a small admixture of states of ordinary quarks and antiquarks, an admixture that may be present in the ψ meson.

The identification of the $SU(3)$ symmetry representation to which the ψ meson pertains was based hitherto on a study of the probability ratio of the decays $\psi \to \rho\pi$ and $\psi \to K\bar{K}^*$ and $\psi \to \bar{K}K^*$,[2,3] and also the ratio of the $\psi \to \pi\pi$ and $\psi \to K\bar{K}$ probabilities, for which, however, there are at present only upper bounds. The preliminary result of these data[1,3] was that the role of the octet state in the decay of ψ into hadrons is appreciable.

The purpose of the present article is to call attention to the fact that the weights of the singlet and octet amplitudes of the ψ-meson decay can be determined also from a comparison of the probabilities of the three-particle decays $\psi \to 3\pi$, $\psi \to K\bar{K}\pi$, and $\psi \to K\bar{K}\eta$, where $\pi^+\pi^-$ do not produce ρ, and $K\pi$ and $\bar{K}\pi$ do not produce K^*.

The spatial part of the matrix element of the decay of the vector meson into three pseudoscalar ones takes the form

$$M = \epsilon_{\mu\nu\lambda\sigma} V_\mu p_{1\nu} p_{2\lambda} p_{3\sigma} T, \tag{1}$$

where V_μ is the vector-meson polarization vector, p_i are the 4-momenta of the pseudoscalar mesons, and T is a function of the invariants $(p_1+p_2)^2$ and $(p_1+p_3)^2$. In our case, when all three pseudoscalar mesons belong to one octet, the unitary part of the matrix element should be completely antisymmetrical relative to permutations of three pseudoscalar mesons, i.e., it should be of the form

$$M_u^s = g_s \sum_{\text{over the permutation}} (-1)^P \phi_k^i(1)\phi_e^k(2)\phi_i^e(3)\psi\sqrt{1/3}\,, \tag{2}$$

if ψ belongs to a unitary singlet and

$$M_u^o = g_o \sum_{\text{over the permutation}} (-1)^P \phi_k^i(1)\phi_e^k(2)\phi_m^e(3)\psi_i^m\,, \tag{3}$$

if ψ is the eighth component of a unitary octet ψ_i^m (normalized to $\psi_i^m\psi_m^i = 1$). With the aid of (2) and (3) we can calculate the amplitudes and the relative probabilities of the decays of ψ into three pseudoscalar mesons. The results are listed in the table, using the following notation: the ψ-meson wave function is

$$\psi = \cos\theta\,|\,1, V\rangle + \sin\theta\,|\,8, V\rangle$$

the wave function of the η meson is

$$\eta = \cos\chi\,|\,8, ps\rangle + \sin\chi\,|\,1, ps\rangle$$

$|1,V\rangle$, $|8,V\rangle$, $|1,ps\rangle$, and $|8,ps\rangle$ are the singlet and octet states of the vector and pseudoscalar mesons, λ is the ratio of the matrix elements of the $\psi \to 3P$ decay in the octet and singlet states, μ is the ratio of the matrix element of the $\psi_8 \to 2P_8\eta_{\text{sing}}$ decay to the matrix element of the $\psi_8 \to 2P_8\eta_{\text{oct}}$ decay [$\mu = 1$ in $SU(6)$ symmetry].

The second row of the table gives the relative probabilities of the decay in the limit of the exact $SU(3)$ symmetry without allowance for the difference between the masses of the π, K, and η mesons, while the third line gives the same quantities for the decay of the virtual γ quantum. (As usual, the γ quantum is described as the state $\frac{2}{3}j_1^1 - \frac{1}{3}j_2^2 - \frac{1}{3}j_3^3 = j_3 + 3^{-1/2}j_8$). The last row of the table lists the ratios of the squares of the matrix elements (1), integrated over

Types of decay	$\pi^+\pi^-\pi^0$	$\eta\pi^+\pi^-$	$K^0K^-\pi^+ = \bar{K}^0K^+\pi^-$	$K^+K^-\pi^0 = \bar{K}^0K^0\pi^0$	$K^+K^-\eta$	$K^0\bar{K}^0\eta$
ψ	$(\cos\theta + \frac{\lambda}{\sqrt{2}}\sin\theta)^2$	0	$\frac{1}{2}\cos^2\theta$	$\frac{1}{4}\cos^2\theta$	$\frac{3}{2}[\cos\chi\cos\theta - \frac{\lambda}{3}\sin\theta(\sqrt{2}\cos\chi - \mu\sin\chi)]^2$	$\frac{3}{2}[\cos\chi\cos\theta - \frac{\lambda}{3}\sin\theta(\sqrt{2}\cos\chi - \mu\sin\chi)]^2$
γ	1	$\frac{1}{3}(\cos\chi + \mu\sqrt{2}\sin\chi)^2$	0	1	$\frac{1}{3}(\cos\chi + \mu\sqrt{2}\sin\chi)^2$	$3\cos^2\chi$
$\int \lvert M\rvert^2 \frac{d^3p}{E}$	2.2	1.25	1	1	0.6	0.6

phase space, for real particle masses (assuming that T is constant, i.e., it does not depend on the particle masses).

As seen from the table, the ratio of the probabilities of the decays $\psi \to \bar{K}K\pi$ to the $\psi \to 3\pi$ probability in the case when ψ is a unitary singlet is $\Gamma(\bar{K}K\pi)/\Gamma(3\pi) = (3/2)/2.2 \approx 0.7$.

If $|M|^2$ is integrated not over all of phase space, but in the region $m_{\pi\pi}^2 = m^2$ for the decay $\psi \to 3\pi$ and $m_{\pi\bar{K}}^2 = m_{K^*}^2$ for the decay $\psi \to K\bar{K}\pi$, then the $\psi \to KK\pi$ and $\psi \to 3\pi$ probability ratio, when at least one combination falls in this region, is only $(3/2)/(3/2 \times 3.1) \approx 0.3$. [The factor (3/2) in the denominator is due to the fact that on the Dalitz plot of the $\psi \to 3P$ decay ρ corresponds to three bands and K^* to two bands.]

The ratio $\Gamma(\bar{K}K^* + \bar{K}^*K)/\Gamma(\rho\pi)$, i.e., the ratio of the same decays when the $K\pi$ masses are in the region K^*, and the 2π masses are in the region of ρ, was determined experimentally.[2] The measured ratio turned out to be $0.4^{+0.2}_{-0.1}$ in comparison with that expected theoretically[1,3] for the unitary singlet $\Gamma(\bar{K}K^* + K^*K)/\Gamma(\rho\pi)_{\text{theor}} = (4/3) \times 0.85 = 1.13$. Since this ratio in the background should be equal to 0.3 for the unitary singlet, it is possible that the experimentally observed ratio $\Gamma(\bar{K}K\pi)/\Gamma(3\pi)$ is due in part to the contribution of the background, and not to admixture of a unitary singlet. It would be of definite interest to check on this circumstance by observing the decays $\psi \to \bar{K}K\pi$ and $\psi \to 3\pi$ in a region outside the resonances of K^* and ρ, as would be a study of the $\psi \to K\bar{K}\eta$ decays. It appears that it would be best to compare experiment in the central regions of the Dalitz diagrams, where one might assume the corrections for the differences between the masses of the final particles to be least essential.

[1]F. J. Gilman, Invited talk presented to the 6th Intern. Conf. on High Energy Phys. and Nucl. Structure, Santa Fe, N. M., June 1975, Preprint SLAC-PUB-1600, 1975.

[2]V. Luth, A. M. Boyarski *et al.*, Report on Intern. Conf. for High Energy Phys., Palermo, Italy, June 1975, Preprint SLAC-PUB-1599, LBL-3397, 1975.

[3]J. L. Rosner, Invited talk at the Annual Meeting of American Physical Society, Seattle, Wash., August 1975, Preprint PRE 19336, 1975.

Erratum: One possibility of explaining the properties of the ψ meson in *SU*(3) symmetry [JETP Lett. 23, No. 3, 160–163 (February 1976)]

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The fraction $\frac{3}{2}$ in the last and penultimate columns in the table should be replaced by $\frac{3}{4}$.