Quantum capillary waves—collective excitations of a Fermi-liquid drop

É. E. Sapershteĭn, S. V. Tolokonnikov, S. A. Fayans, and V. A. Khodel'

V. I. Kurchatov Institute of Atomic Energy
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It is shown that low-lying collective states of atomic nuclei belong to a new branch of collective excitations of a Fermi-liquid drop, namely quantum capillary waves.

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It was shown in[1] that at $T=0$ a Fermi-liquid drop contains, besides zero-sound oscillations, one more branch of collective excitations, viz., quantum capillary waves (capons). These excitations are similar in many respects to the usual classical surface oscillations of the drop, but at small angular momenta $L \sim 1$ quantum effects become strongly pronounced in them. Observation of these effects in ^{3}He is possible when the quasiparticle mean free path becomes comparable with the drop radius R, but this calls for too low temperatures ($T \lesssim 10^{-4}$ °K at $R=1$ cm). The most suitable object for the investigation of quantum capillary waves are therefore atomic nuclei.

We calculate in this paper the characteristics of a number of low-lying collective states (LCS) of nuclei, and show that they can be interpreted as quantum capillary waves. The properties of the LCS are determined by the transition amplitude g_L, which satisfies the equation[2]

$$g_L^i(r;\omega) = \int F_L^{ik}(r, r')A_L^k(r', r''; \omega)g_L^k(r'',\omega)dr'dr''. \qquad (1)$$

Here F_L and A_L are harmonics, in the angle between r and r', of the amplitude of the interaction of the quasiparticles F and the particle-hole propagator A, and $d\tau = r^2 dr$:

$$A_L(r, r'; \omega) = \int G(r, r'; \epsilon + \frac{\omega}{2})G(r', r; \epsilon - \frac{\omega}{2})P_L(nn')\frac{d\epsilon}{2\pi i}\frac{dn\,dn'}{4\pi}$$

$$= \sum_{nljl'j'} C_L^{ljl'j'} n_{nlj} R_{nlj}(r)R_{nlj}(r')[G_{l'j'}(r, r'; \epsilon_{nlj} + \omega) \qquad (2)$$

$$+ G_{l'j'}(r, r'; \epsilon_{nlj} - \omega)],$$

where $C_L^{jj'j'}$ is an angle coefficient containing $3j$ and $6j$ symbols, R_{nlj} are the radial single-particle wave functions, and n_{nlj} are the quasiparticle occupation numbers. The Green's function of the quasiparticle is

$$G_{lj}(r, r'; \epsilon) = \frac{1}{rr' W_{lj}(\epsilon)} \gamma_{1lj}(r_<; \epsilon)\, \gamma_{2lj}(r_>; \epsilon), \tag{3}$$

γ_1 and γ_2 are independent solutions of the radial Schrödinger equation

$$\left(-\frac{1}{2m^*}\frac{\partial^2}{\partial r^2} + U_{lj}(r)\right)\gamma_{lj}(r;\epsilon) = \epsilon\, \gamma_{lj}(r;\epsilon)$$

with a self-consistent potential V_{lj}, and W_{lj} is the Wronskian. In contrast to the standard method, calculation of A in the coordinate representation in accord with formulas (2) and (3) takes into account automatically the entire continuous spectrum of the single-particle states. This method was proposed in[3] and later independently in[4].

Knowing the amplitude g_L, we can find the density matrix of the transition $\rho_{0s}(\mathbf{r}, \mathbf{r}')$, the transition density $\nu_{0s}(\mathbf{r}) = \rho_{0s}(\mathbf{r}, \mathbf{r})$, and the transition current

$$\mathbf{j}_{os}(\mathbf{r}) = \rho(\mathbf{r})\mathbf{v}_{os}(\mathbf{r}) = \frac{1}{2m^* i}\left[\left(\frac{\partial}{\partial \mathbf{r}} - \frac{\partial}{\partial \mathbf{r}'}\right)\rho_{os}(\mathbf{r}, \mathbf{r}')\right]_{\mathbf{r}\to\mathbf{r}'} \tag{4}$$

(here $\rho(\mathbf{r})$ is the quasiparticle density); $B(EL\uparrow) = (2L+1)\left[\int_r^L \nu_L(r)\, dr\right]^2$.

The calculation method is based on a consecutive allowance in Eq. (1) of the conditions for the self-consistency[5,6] between the effective interaction F and the self-consistent nuclear potential U

$$\frac{\partial U^i}{\partial r} = \int F_1^{ik}(r, r')A_1^k(r', r''; \omega = 0)\frac{\partial U^k}{\partial r''}\, dr'\, dr''. \tag{5}$$

From a comparison of (1) and (5) we see that the frequency of the dipole oscillations of the center of gravity is $\omega_1 = 0$, with $g_1(r; 0) \sim \partial U/\partial r$. It is just these properties that distinguish the Fermi-liquid drop (a system with spontaneous violation of translational symmetry) from a system of Fermi particles locked in a box. By virtue of the general theorems[7] this violation is the cause of the appearance of a new branch of collective excitations. In the crystal this branch is made up by phonons, and in a finite Fermi system by quantum capillary waves. The details of the calculation of their characteristics are given in[8]. In the calculations one uses an amplitude F with a Gaussian radial dependence

$$F(r, r') = C(\sqrt{2\pi} r_G)^{-3} \exp[-(r-r')^2/2r_G^2]\left[\hat{f}_{ex} + (\hat{f}_{in} - \hat{f}_{ex})\xi\left(\frac{r+r}{2}\right)\right], \tag{6}$$

where $\hat{f} = f + f'\tilde{\tau}_1\tilde{\tau}_2$, $\xi(r) = [1 + \exp((r - R_{int})/a)]^{-1}$, $C = 360$ MeV-F^3, $r_g = 1$ F, $R_{int} = 7$ F, $a = 0.67$ F, $f_{in} = 0.26$, $f_{ex} = -2.54$, $f'_{in} = f'_{ex} = 0.65$. This interaction agrees with the self-consistency condition (5) with an accuracy of the order of 10%. The parameters of the self-consistent field were taken from[9].

The table and Figs. 1—3 show only the results of the calculations for ^{208}Pb and ^{40}Ca.

TABLE I. Characteristics of low-lying collective states of the nucleus ^{208}Pb

State	C_L/C_L^{hydr}	B_L/B_1^{hydr}	ω_L, MeV	$B(EL\uparrow)$ (in Weisskopf units)	$B(EL\uparrow)^{exp}$ [10]
3^- (2.61 MeV)	1.95	3.55	2.56	39	39.5 ± 2.0
5^- (3.20 MeV)	1.68	5.20	3.18	13	14 ± 5
2^+ (4.08 MeV)	23.6	1.89	4.23	8	8.1 ± 0.5

The results show that the collective motion in a quantum capillary wave is similar in many respects to the classical motion—in particular, the transition amplitudes g_L and the transition densities ν_L have a sharp surface peak (Figs. 1 and 2; $g_L^{cl} \sim \partial U/\partial r$, $\nu_L^{cl} \sim \partial\rho/\partial r$). But it differs in two essential features. First, it is vortical, and consequently the mass coefficients B_L do not coincide with the hydrodynamic coefficients B_L^{hydr} (Fig. 3 shows the radial distribution of the velocity curl for the state 3^- of ^{208}PB; curl $v_{L,M}^t(\mathbf{r}) = \beta_L R_L^t(r) Y_{LM}(\mathbf{n})$, where β_L is the velocity of the nuclear boundary). Second, an important role is played in it by compressibility effects, which influence particularly strongly the rigidities C_L. These effects are also responsible for the appearance of exchange corrections to the classical surface peak in the transition densities.

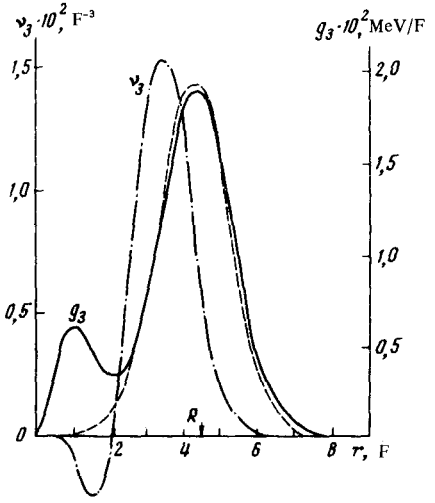

FIG. 1. Transition amplitude and transition density for the state $3^-(3.73)$ MeV in ^{40}Ca. The dashed line shows the derivative $\partial U^p/\partial r$.

Although the transition densities of all low-lying states all have peaks of hydrodynamic origin, one must not assume that all are quantum capillary waves. Analysis shows that some of these states constitute superpositions of quantum capillary waves with individual particle-hole configurations (e.g., the first 2^+ state in ^{208}Pb). One should regard as purely collective those excitations whose density matrices do not contain a preferred contribution of any one particle-hole configuration—this statement pertains, of course, also to quantum

capillary waves. From this point of view, the purest quantum capillary waves are the first 3⁻ states in even-even nuclei.

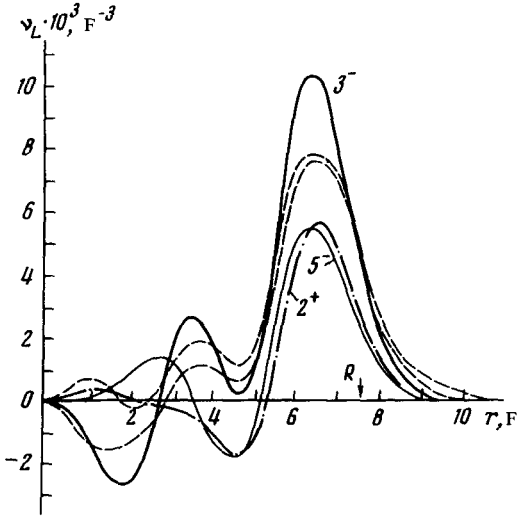

FIG. 2. Charged components of the transition densities for the lowest states in ^{208}Pb. Dashed-fit[11] to the experimental scattering of electrons with excitation of the 3⁻ level.

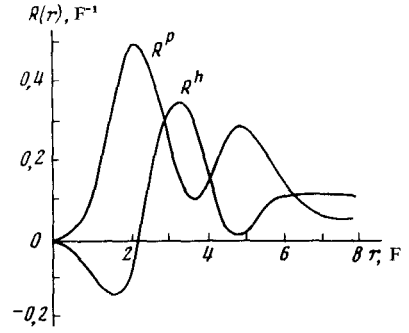

FIG. 3. Radial distribution of the velocity curlv_{0s}(**r**) for the 3⁻ state in ^{208}Pb.

Thus, collective excitations analogous to classical capillary waves exist also at $T=0$, i.e., in the "antihydrodynamic" limit, when $\omega\tau \gg 1$ (ω is the excitation frequency and τ is the quasiparticle lifetime). It is of interest to organize an experiment aimed at searching for quantum capillary waves in ^{3}He, which is the only macroscopic Fermi system in which they can exist.

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