A nuclear analog of the Cerenkov effect

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We estimate the angular and energy distributions, as well as the total probability of "spilling" of particles out of the nucleus by a shock wave generated in Cerenkov fashion in collisions of high-energy heavy ions.

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It is shown in[1] that the high-frequency component of the effective field $V_\omega(\mathbf{x})$, describing giant resonances of a nucleus, satisfies an equation of the hydrodynamic type (G is the internucleon interaction and ρ is the density of the nucleus)

$$-\omega^2 V_\omega(\mathbf{x}) = \int d\mathbf{x}\, G(\mathbf{x}\,|\,\mathbf{x}')\,\mathrm{div}\left(\rho(\mathbf{x}')\frac{\partial V_\omega}{\partial \mathbf{x}'}\right) - \omega^2 \overset{\circ}{V}_\omega(\mathbf{x}) \qquad (1)$$

and a "Cerenkov" mechanism for the generation of the shock wave in the nucleus was proposed.[1)]

Assuming the motion of the incident particle to be given, we have for the external field $V(\mathbf{x}, t)$

$$\overset{\circ}{V}(\mathbf{x}, t) = \int d\mathbf{x}' G(\mathbf{x} | \mathbf{x}') \, \delta\rho(\mathbf{x}', t), \tag{2}$$

where $\delta\rho(\mathbf{x}, t)$ is the external perturbation of the nuclear density. This approximation is macroscopic in character and is valid only when the perturbation $\delta\rho$ affects a sufficiently large number of particles, i.e., for the collision of heavy ions.

A solution of (1) can be obtained by expanding $V(\mathbf{x})$ in the eigenfunctions $V_\alpha(\mathbf{x})$ of the homogeneous equation[1] (with $\overset{\circ}{V}=0$)

$$V_\omega(\mathbf{x}) = \overset{\circ}{V}_\omega(\mathbf{x}) + \sum_a \frac{\omega_a^2}{\omega^2 - \omega_a^2} V_a(\mathbf{x}) \, Tr\,(u_a^+ \overset{\circ}{V}_\omega). \tag{3}$$

It is easy to show that for a zero force radius G and a perturbation $\delta\rho$, the field $V(\mathbf{x}, t)$ is discontinuous and becomes infinite on the "Cerenkov" cone, i.e., it describes a density shock wave. The shock wave "spills out"[3] the particles of the nucleus into the continuous spectrum with a probability $W_{0\to E}$

$$W_{0\to E} = \left| \int_{-\infty}^{\infty} dt \, e^{i(E-E_0)t} (E | V(\mathbf{x}, t) | 0) \right|^2. \tag{4}$$

The energy and angular distributions can be obtained from qualitative considerations. The shock-wave front, moving with the speed of sound c, throws out the nuclear particles predominantly along the normal to the front, and with velocities close to $2c$. In addition, the probability of "spilling" (4) will be proportional to the area of the cone (for the nucleus, $\sim A^{2/3}$). The deviations from this classical picture are connection with the Fermi motion of the nucleons, and also with the finite radius of the perturbation $\delta\rho$ and of the interaction G.

Bearing in mind the volume character of the effect, let us estimate (4) for a homogeneous Fermi liquid, we put $G(\mathbf{x}|\mathbf{x}') \sim \delta(\mathbf{x}-\mathbf{x}')$ and $\delta\rho(\mathbf{x}, t) \sim \delta(\mathbf{x}-\mathbf{v}t)$, where $\mathbf{v}$ is the velocity of the external perturbation. The kinematic singularities of W in this case will be most strongly pronounced. The probability of production of a particle with momentum $\mathbf{p}$ takes the form

$$\frac{dW}{dp} = \frac{\pi^3 f^2 T^2}{16 p_F^2} \sum_{\mathbf{p}'} n_{\mathbf{p}'} (kc)^3 \delta(\omega - \mathbf{k}\mathbf{v}) \delta(\omega^2 - k^2 c^2), \tag{5}$$

where $\mathbf{k}=\mathbf{p}-\mathbf{p}'$; $\omega=\omega_{\mathbf{p}}-\omega_{\mathbf{p}'}$; $c=p_F\sqrt{f/3}$; f is the scalar scattering amplitude, $n_{\mathbf{p}}$ is the occupation number, and $T \sim R/v$ is the time of the process. The factor T^2 reflects the two-step character of the effect—emission of sound followed by production of a particle-hole pair.

The results of straightforward but cumbersone integrations in (5) are represented in Figs. 1 and 2 by the energy $[\partial W(p)/\partial p]$ and angular $[\partial W(\theta)/\partial\theta]$ distributions, where $\theta_0=\cos^{-1}(c/v)$ is the Cerenkov angle, and $\cos\theta=\mathbf{p}\cdot\mathbf{v}/pv$. The parameter $p_F/2c$ characterizes the influence of the Fermi motion of the nucleon on $W(p, \theta)$. The values of the functions on the figures are given in relative units; in addition, $(c/v)^3$ has been factored out. We note the good agreement of the results of a direct calculation of W with the classical picture of the "spilling out" of the particles by the front of the shock wave. Attention is called to the universal dependence of the distributions on $v/c[(1/v^3)f(\cos\theta_0)]$.

To estimate the absolute value of W we need a more detailed analysis. The

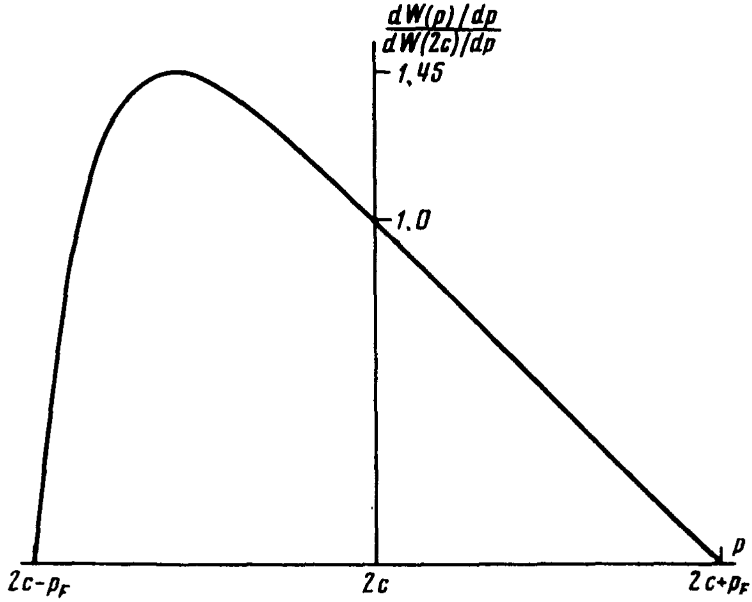

FIG. 1. Momentum distributions, integrated over the angles, of the particles "spilled out" by the shock wave: $p_F/2c=\frac{1}{3}$.

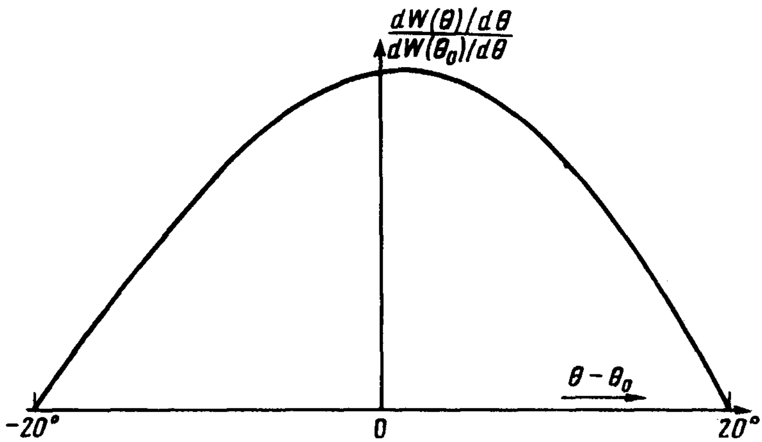

FIG. 2. Angular distributions of the secondary particles: $v/c=1.5$; $p_F/2c=\frac{1}{3}$.

main contribution to the particle radiation is introduced by the region of the front (where in our approximation $V=\infty$). Therefore, the value of W is sensitive to the parameters that cut off the high-frequency part of the spectrum of the natural frequencies and the harmonics generated by the perturbation $\delta\rho(\mathbf{x}, t)$. Exerting practically no influence on the form of the distribution

$W(\theta, \mathbf{p})$ ($\mathbf{p} \approx 2c$; $\theta \sim \theta_0$), the cutoff strongly decreases the integral probability. Approximating $\delta\rho(\mathbf{x}, t)$ by a sphere of constant density and taking into account the dispersion of the interaction G, which forbids the propagation of too short waves (with $\omega_\alpha/c > p_F$), we obtain

$$W = \int \frac{dW}{d\mathbf{p}} d\mathbf{p} \approx 0.05(AA')^{2/3}(c/v)^3,$$

where A' is the atomic number of the incident ion. Thus, the probability W is high and for reliable quantitative estimates it is necessary to take into account the multiple excitation.

In the final nucleus it is necessary to expect the additional smearing of $W(p;\theta)$ due to reflections from the walls and the damping of the giant resonances. In addition, it is necessary to separate in (1) the dipole branch of the excitations, corresponding to motion of the nucleus as a whole. A standard procedure leads to an additional term in the right-hand side of (1)

$$-\int d\mathbf{x}' G(\mathbf{x}|\mathbf{x}') \operatorname{div} \rho(\mathbf{x}') < \frac{\partial V_\omega}{\partial \mathbf{x}} > ,$$

where

$$< \frac{\partial V_\omega}{\partial \mathbf{x}} > = 1/A \int \rho(\mathbf{x}) \frac{\partial V_\omega}{\partial \mathbf{x}} d\mathbf{x} .$$

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[1)]We note that the region of applicability of ordinary hydrodynamics in a real nucleus is highly restricted. Simple estimates[2] for the nuclear excitation energy yield $E^* \gg \omega A$ and $E^* \gg \epsilon_F A^{2/3}$ (ω is the frequency of the sound, A is the atomic number, $\epsilon_F = p_F^2/2$ is the Fermi energy, and $\hbar = m = 1$). Even if these inequalities are satisfied, the question of the possibility of rapid relaxation of the nucleus to a state of local statistical equilibrium at such large E^* still remains open.

[2)]We must bear in mind here that the integral operator in (1) is non-Hermitian, and we must use the associated set of eigenfunctions $U_\alpha(x)$

$$\left(-\omega_\alpha^2 U_\alpha(\mathbf{x}) = \operatorname{div}(\rho(\mathbf{x}) \frac{\partial}{\partial \mathbf{x}} \int G(\mathbf{x}|\mathbf{x}') U_\alpha(\mathbf{x}') d\mathbf{x}')\right).$$

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