Dynamic similarity at critical points of arbitrary order

V. V. Prodnikov and G. B. Teĭtel'baum

Kazan' Physico-technical Institute, USSR Academy of Sciences
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The dynamic exponents at the critical points of higher order are determined and their dependence on the conservation law is investigated.

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Systems with critical points, which are simultaneously critical for several phases, are presently intensively investigated. It is the number σ of these phases which determine the order of a critical point. At $\sigma>2$, the second-order phase transition curves go over in such points into first-order phase transition curves. Such a behavior of the curves is possible for the mixture ^3He—^4He, for metamagnets, and for compressible magnets.[1–5] A significant change in the static critical properties has been observed as a function of the number σ of phases near the critical point. A change takes place also in the dimensionality d_σ of space starting with which a stable nontrivial solution appears for the renormalization group equations for the coupling constants of the fluctuating fields.[2,6] Expansion in powers of the deviation $\epsilon_\sigma=d_\sigma-d$, of the dimensionality d of the system from $d_\sigma=2\sigma/(\sigma-1)$ makes it possible to calculate the resultant deviations of the critical exponents from the predictions of the mean-field theory.[7–9] In this paper this approach is generalized for the investigation of the dynamics at a critical point of arbitrary order. Let the Hamiltonian of the system be a Ginzburg-Landau functional

$$\mathcal{H}(\vec{\psi}) = \int d^d x \left[\frac{1}{2} \mid \nabla \vec{\psi}(\mathbf{x}) \mid^2 + \sum_{k=1}^{\sigma} \frac{u_{2k}}{(2k)!} (\vec{\psi}\vec{\psi})^k - \mathbf{h}\vec{\psi}\right] \quad (1)$$

with a single n-component order parameter $\vec{\psi}(\mathbf{x})$, where $\mathbf{h}(\mathbf{x},t)$ is the field conjugate to it. The dynamic equation can be obtained from the condition that the rate of the change of the order parameter be proportional to the conjugate thermodynamic force. The proportionality coefficient Γ_0, which sets the time scale, plays the role of the nonrenormalized kinetic coefficient and depends on the law governing the conservation of the order parameter. The action of the thermal reservoir is taken into account by introducing a random Gaussian force with spectral intensity $2\Gamma_0$. The gauge invariance of the equation of motion makes it possible to represent, independently of the number σ of the phases, the correlation function in the dynamic-similarity form

$$G^{-1}(q,\omega) = q^{2-\eta_\sigma} f(\omega/\omega_q), \qquad \omega_q \sim q^{z_\sigma}, \tag{2}$$

where η_σ is the static exponent and z_σ the dynamic exponent at the critical point, q is the wave vector, and ω is the frequency.

To find the dynamic exponent z_σ, we write the correlator in the form of the following expansion

$$G^{-1}(q,\omega) = G_0^{-1}(q,\omega) + \Sigma(q,\omega) - \Sigma(0,0), \tag{3}$$

where $G_0(q,\omega) = [q^2 - i\omega/\Gamma_0]^{-1}$ is the nonrenormalized correlator, $\Sigma(q,\omega)$ is the self-energy part, and the subtraction $\Sigma(0,0)$ includes all the corrections that do not depend on the frequency or momentum. The nonrenormalized vertices in the expansion should be u_{2k}, which leads to the appearance of closed loops that do not depend on q or ω. It is therefore convenient to go over directly to new vertices $\bar{u}_{2k}$, which include these loops (Fig. 1). The second-order corrections, with which the expansion of the self-energy part begins, are then given by diagrams with $2k-1$ internal lines (Fig. 2). The dynamic fluctuations

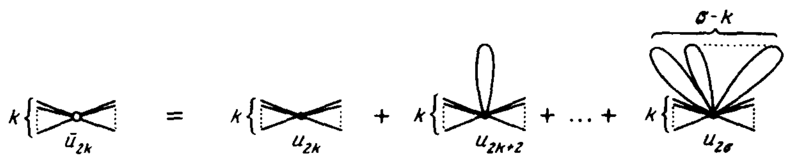

FIG. 1.

lead to replacement of the nonrenormalized vertices by renormalized ones $\tilde{u}_{2k}$, which in principle can depend on the frequency. These satisfy the following dimensional estimates

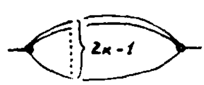

FIG. 2.

$$\tilde{u}_{2k} \sim \omega^{[\epsilon - d_\sigma + 2k(k-1)^{-1}](k-1)/2}, \tag{4}$$

from which it follows that for $k < \sigma$ all $\tilde{u}_{2k} = 0$. Comparison of (4) with the expansion $\tilde{u}_{2\sigma}$ in powers of small $\bar{u}_{2k}$ yields in first order $\tilde{u}_{2\sigma} \sim \epsilon_\sigma$. Therefore, in the expression for $\Sigma(q,\omega)$, accurate to $O(\epsilon_\sigma^2)$, we can confine ourselves to second-order diagrams with $k=\sigma$, and for the renormalized coupling constants we can use their static values obtained on the basis of,[8,9] $\tilde{u}_{2\sigma}=\epsilon\sigma(2\pi)^{\sigma}(2\sigma)!g_{\sigma}^{-1}\times(n)\Gamma^{1-\sigma}(\nu)$, where $\nu=(\sigma-1)^{-1}$ and

$$g_{\sigma}(n) = \sum_{j} \frac{(\sigma!)^2\left(\frac{n}{2}+\sigma-1\right)!\,(2\sigma-2j)!}{(\sigma-2j)!\,(j!)^2\left(\frac{n}{2}+\sigma-j-1\right)!\,(1+\sigma)!} . \tag{5}$$

It follows therefore that at $d \geq d_{\sigma}$ the dynamics is described within the framework of the average-field theory, and fluctuation corrections can arise when $d<d$.

Let us analyze a number of concrete cases corresponding to different conservation laws in the system. Let the order parameter not be conserved, i.e., $\Gamma_0=\text{const}$. Calculation of the diagrams and allowance for the combinatorial factors yields as $\omega\to 0$

$$\Sigma^{(2)}(q=0,\omega) = i a_{\sigma}\eta_{\sigma}\Gamma_{0}^{-1}\,\omega\ln\omega, \tag{6}$$

where

$$a_{\sigma} = \frac{1}{2}\,d_{\sigma}(2\sigma-1)\Gamma^{1-2\sigma}(\nu)\int_{0}^{\infty} dx\,x^{d_{\sigma}-5}e^{-x^2}\gamma^{2(\sigma-1)}(\nu,x^2),$$

$\gamma(\nu x^2)$ is the incomplete Gamma function

$$\eta_{\sigma} = 2\epsilon_{\sigma}^{2}(\sigma-1)^{2}\sigma!\left(\frac{n}{2}+\sigma-1\right)!\left[\left(\frac{1}{2}n\right)!\,g_{\sigma}^{2}(n)\right]^{-1}.$$

Comparison of (3) and (6) with formula (2) leads to

$$z_{\sigma} = 2 + c_{\sigma}\eta_{\sigma}, \quad c_{\sigma} = 2a_{\sigma}-1, \quad c_{\sigma}>0. \tag{7}$$

From this it follows, in particular, that the renormalized kinetic coefficient $\Gamma_q \sim \omega_q\chi_q \sim q^{2a_{\sigma}\eta_{\sigma}}$ vanishes as $q\to 0$ at the transition point. We note that at $\sigma>2$ our results yield the fluctuation corrections in flat models. With increasing σ, their magnitude decreases rapidly. In the limit as $n\to\infty$ we have $(z_s-2)\sim 1/n$ at even σ and $(z_{\sigma}-2)\to\text{const}\neq 0$ at odd σ. A numerical calculation yields $c_2 = 6\ln(4/3)-1\approx 0.726$, $c_3\approx 0.946$, $c_4\approx 1.053$, and $c_5\approx 1.118$. For the usual critical point ($\sigma=2$) our result agrees with that of[10].

If the order parameter is a conserved quantity, then $\Gamma_0\sim q^2$ and no terms of the type $\omega q^{-2}\ln\omega$ appear in $\epsilon(q,\omega)$. Consequently, in this case we arrive at the result $z_{\sigma}=4-\eta_{\sigma}$ of the Van Hove theory.

In degenerate systems, precession of the conserved order parameter can take place. Allowance for this motion makes the Liouville operator non-Hermitian. The corresponding elaboration of the equations of motion,[11] carried out for a model with a three-component order parameter, and a subsequent dimensional analysis, yield $z_{\sigma}=1+d/2$. We note that the dynamic exponent does not depend here on the number of phases surrounding the critical point, as is apparently characteristic of models with a non-Hermitian Liouville operator.

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