Singularities of Shubnikov-de Haas effect in thin antimony plates

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We investigate the galvanomagnetic properties of platelike single crystals (whiskers) of antimony. Two types of resistance oscillations are observed. The first is observed in strong magnetic fields $2r < d$ and constitutes the Shubnikov–de Haas (SdH) effect on the extremal section of the hole Fermi surface (r is the radius of the orbit). The second is observed in weak fields $2r > d$, and analysis of the experimental results show that it is the SdH effect on the same extremal section truncated by the surfaces of the sample. This phenomenon was predicted by Kosevich and Lifshitz in 1955.

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We investigated the influence of the dimensions on the galvanomagnetic properties of antimony. The measurement objects were chosen to be thin single-crystal plates of antimony (whiskers), which were grown by us from the gas phase.[2] The thickness d of the investigated samples, determined with the aid of an interference microscope with accuracy not worse than 10%, ranged from 0.8 to 0.08 μ. The planes of the sample plates were parallel to the basal plane, and a *dc* measuring current I was made to flow along the binary axis.

We measured the resistance R and its derivative $\partial R/\partial H$ as functions of the magnetic field H. The measurements were performed in fields up to 80 kOe at a temperature $T = 4.2$–1.3 °K. In all the investigated 11 samples, resistance oscillations of a new type were observed, substantially different from the usual Shubnikov—de Haas (SdH) effect.

The results are illustrated by automatic recorder plots shown in Figs. 1 and 2 for three samples, for which the magnetic field H was parallel to the surface of the plates and to the bisector axis, and the current $\mathbf{I} \perp \mathbf{H}$. In Fig. 1, for sample $Sb-1$ ($d = 0.14$ μ) one can distinguish two singularities near 27 and 33 kOe against the background of the monotonic $R(H)$ dependence.[1)] The SdH effect is practically unnoticeable in this scale, up to fields 60 kOe. If this $R(H)$ depen-

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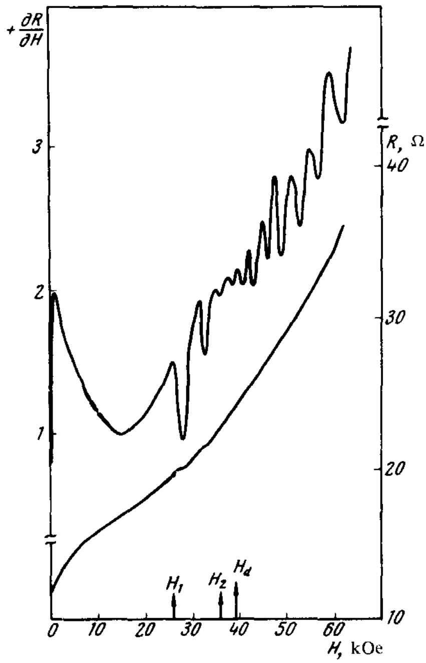

FIG. 1. Dependence of the resistance and of its derivative on the magnetic field for the sample $Sb-1$ at $T=1.3$ °K. Sample dimensions: length 212 μ, width 6.4 μ, thickness 0.14 μ. The field is parallel to the plane of the sample and to the bisector axis. The values of H_1, H_2, and H_d were calculated in accordance with the formulas of the text.

dence is differentiated, then oscillations are observed in the region of strong fields, which are unambiguously interpreted as the SdH effect with a period 1.4×10^{-6} Oe^{-1}. Measurements along various directions of **H** have shown that this period pertains to the extremal section of one of the three hole Fermi surfaces of antimony. This section can be regarded, with acceptable error, as an ellipse with semi-axes $p_2=4.3\times10^{-21}$ g-cm-sec^{-1} and $p_2=5.7\times10^{-21}$ g-cm-sec^{-1} ($p_{2,3}$ is the Fermi momentum in the direction of the principal axes).[3,4]

In weak fields, the character of the oscillations is fundamentally different: the amplitudes of the peaks increase with decreasing field, and the distance between peaks increases. The number of oscillation peaks is limited and is small in comparison with the SdH effect, while the first peak for thin samples appears in relatively strong fields, (for example, in a field $H\approx 50$ kOe for the sample Sb − 3 ($d=0.08$ μ)).

With increasing thickness d, the picture of the behavior of the oscillations remains qualitatively the same. However, the region where they exist shifts towards weaker fields, and the number of peaks increases (Fig. 2). At the same time, the SdH oscillations appear in weaker fields.

The new type of oscillations is observed with the field both transverse and longitudinal relative to the current **I**. If the field **H** is perpendicular to the plane of the plate, then the oscillations of the new type do not appear, and their place is occupied by the usual SdH effect with a period 1×10^{-6} Oe^{-1}.

There is no doubt that the new phenomenon is a size effect. Indeed, if we calculate the field $H_d=2p_2c/ed$ at which the orbit diameter is equal to the sample thickness, then the field H_d for the extremal section considered above coincides

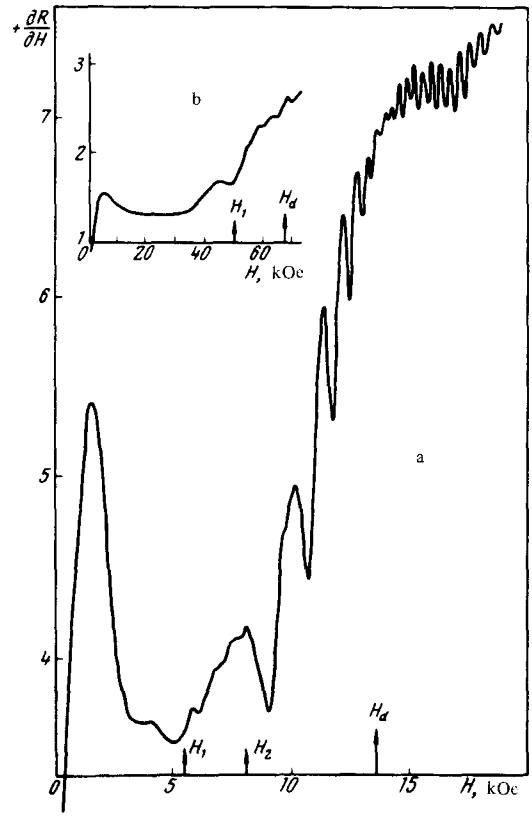

FIG. 2. Derivatives of the resistance vs. the magnetic field: a) $Sb-2$, length 140 μ, width 5 μ, thickness 0.4 μ, $T=1.3$°K; b) $Sb-3$, length 230 μ, width 7 μ, thickness 0.08 μ, $T=4.2$°K.

with the boundary that separates the regions of the existence of the oscillations of the two types (see Figs. 1 and 2). Therefore the new phenomenon should be associated with quantization under conditions when the orbit is bounded by the minimal dimensions of the sample, and the ordinary SdH effect cannot exist. Three types of quantization have been predicted under the conditions $2r>d$. The cause of the appearance of the singularities of the resistance in all three cases is the same as in the SdH, namely, the change of the number of the Landau levels under the Fermi level, and as a consequence, the change in the density of the states at the Fermi level.

It is shown in[5] that the singularities of the resistance can arise when the quantity eHd/c becomes comparable with the diameters of the non-extremal sections of the Fermi surface, corresponding to discrete quantum values of the momentum along the field. This phenomenon should exist in samples of round or rectangular cross section, and the character of the electron reflection from the sample surface is immaterial. The distance ΔH between the peaks of the resulting singularities of the resistance W should not depend on the field and is determined by sample thickness: $\Delta H \propto d^{-2}$. Therefore the number of resistance singularities is proportional to the magnetic field $n \propto H$. In the usual SdH effect we have $n \propto 1/H$. (Here and below, n is simultaneously identical to the number of Landau levels in the field H and is an integer.) It is therefore easy to show that the total number of the peaks of the singularity of the resistance in the interval $0 < H < H_d$ should be equal to the total number of peaks in the SdH on the same orbits in the interval $H_d < H < \infty$. This is clearly seen in Fig. 3. It is perfectly obvious that the new phenomenon observed by us has no connection with that considered above.

Two other types of oscillations can exist also in plates in the case of specular reflection of the electrons from the surface. The resistance oscillations should then be connected with the extremal orbits that are truncated by one or two surfaces of the sample. Let us consider these types.

Reflection of the electrons from one surface gives rise to a system of n quantum surface levels corresponding to a discrete set of "jumping" trajectories. These levels were discovered in[6]. In[7] it was suggested that in the case when the height of the arc of the trajectory Z_n is equal to the thickness of the plate d, a singularity appears in the density of states, and accordingly in the resistance. Since the height of the arc in the weak-field region $H \ll H_d$ is $Z_n \propto H^{-1/3} n^{2/3}$, it follows from the equality $Z_n = d$ that in fields $0 < H \ll H_d$ the number of quantum levels and accordingly the number of resistance peaks is $n \sim \sqrt{H}$. Obviously, the distance between peaks increases with the field $\Delta H \sim \sqrt{H}$, and the total number of peaks up to $H = H_d$ is also equal, just as in the first considered case, to the total number of peaks in the SdH effect in the interval $H_d < H < \infty$ (see Fig. 3).

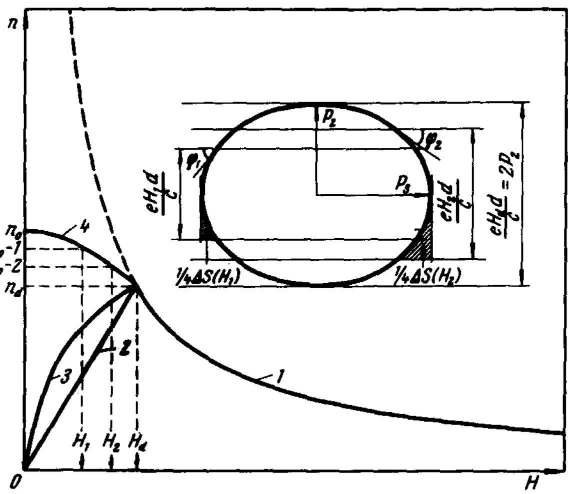

FIG. 3. Dependence of the number of quantum levels for three possible types of size effects for elliptic Fermi-surface sections: 1—ordinary Shubnikov—de Haas effects; $n \propto 1/H$; 2—size effect on non-extremal orbits, $n \propto H$; 3—size effect on orbits truncated by one surface. In weak fields $n \propto \sqrt{H}$, 4—size effect on orbits truncated by two surfaces. In weak fields $n_0 - n \propto H^2$.

This type of quantization also does not conform to the experimental results.

Quantization in the case when the orbit is truncated by two surfaces was considered in[1], where it was shown in general form that the distance ΔH between the singularities should depend in a complicated manner on the magnetic field, on the thickness d, and on the shape of the extremal section of the Fermi surface.

We shall show that the unusual resistance oscillations belong precisely to this type. To this end, we find the number of quantum levels as $H \to 0$. (We neglect the smearing of the levels. If we disregard the sample dimensions, $n \to \infty$). In weak fields the extremal area of the truncated extremal orbit S^d_{ext} can always be represented in the form of a rectangle with sides eHd/c and $2p_3$ (see Fig. 3): $S^d_{ext} \approx 2p_3eHd/c = S_{n_0}(H)$.

On the other hand, in accordance with the quantization conditions, $S^d_{ext} = (2\pi\hbar eH/c)n$. Equating these expressions, we get

$$n(H) \to n_0 = \left[\frac{p_3 d}{\pi\hbar}\right]^{2)}, \quad \text{as} \quad H \to 0. \tag{1}$$

We now obtain the number n_d of the quantum levels in the field H_d. It can be determined in terms of the field H_d and the period of the SdH oscillations $\Delta(1/H) = 2\pi\hbar e/cS_{ext}$:

$$n_d = \left[\frac{1}{H_d\Delta(1/H)}\right] = \left[\frac{S_{ext}d}{4\pi\hbar p_2}\right]. \tag{2}$$

Depending on the shape of the non-truncated extremal section S_{ext}, three cases are possible: $n_0 < n_d$, $n_0 = n_d$, and $n_0 < n_d$.

Thus, the maximum number of levels that can arise (or vanish) for a limited area S^d_{ext} in the interval $0 \leq H \leq H_d$ is equal to $\Delta n = |n_0 - n_d|$. This should also be equal to the number of resistance-oscillation peaks in a field $H < H_d$.

Let us examine the extremal section of the hole "ellipsoid" of the Fermi surface of antimony, corresponding to the observed SdH effect. Using expressions (1) and (2) and the value of the area $S_{ext} = \pi p_2 p_3$, we get:

$$\Delta n = \left[\frac{p_3 d}{\pi\hbar}\right] - \left[\frac{p_3 d}{4\hbar}\right] \approx \frac{n_d}{4}; \qquad n_0 > n_d. \tag{3}$$

Let us compare the results with experiment. Assuming the field separating the regions of the existence of the two types of oscillations to be equal to H_d, and using the value of the period of the SdH oscillations, we can obtain Δn and n_d directly from experiment. The experimental results satisfy relations (2) and (3). Thus, for example, for $Sb-1$ calculation yields $\Delta n = 4$ (or 5) and $n_d \approx 18$; for $Sb-2$ we get $\Delta n = 9$ (or 10) and $n_d \approx 45$.

The positions of the oscillation peaks at $H < H_d$ can be calculated if we know the details of the shape of the section. But we confine ourselves to an approximation calculation of the positions of the first and second peaks on the side of weaker fields. This calculation corresponds to the accuracy with which the true cross section can be represented by an ellipse.

It is physically clear that the first peak rises at the instant when the number of levels n_0 changes by unity. Since the number of levels is $n_0 = cS_{n_0}(H)/2\pi\hbar eH$, and the number of levels in the field H is equal to $n(H) = cS^d_{ext}(H)/2\pi\hbar eH$, it follows that

$$n_0 - n(H) = \frac{c}{2\pi\hbar eH}\left(S_{n_0}(H) - S^d_{ext}(H)\right) = \frac{c\,\Delta S_{ext}}{2\pi\hbar eH}\,. \tag{4}$$

In weak fields we can assume that the extremal section S^d_{ext} is bounded by two straight lines located at distances $\pm eHd/2c$ from the center, and two circles of radius p_3 (Fig. 3). Then, accurate to third order in H, the area difference ΔS_{ext} can be written in the form

$$\Delta S_{ext} \approx 4\left(\frac{eHd}{2c}\right)^3 \Big/ 6p_3\,. \tag{5}$$

Using expressions (4) and (5) and the condition $n_0 - n(H) = 1$, we obtain the field H_1 corresponding to the first peak of the oscillations:

$$H_1 \approx \frac{5c}{e}\sqrt{\frac{\pi\hbar p_3}{d^3}}\,. \tag{6}$$

We can show analogously that the field of the second peak, corresponding to $n_0 - n(H) = 2$, is equal to[3] $H_2 \approx H_1\sqrt{2}$. The calculated values of the fields H_1 and H_2 are in satisfactory agreement with the experimentally observed ones (Figs. 1 and 2).

Less clear is the question of the amplitude of observed peaks, since there is no complete theory of the effect. From qualitative considerations the following is clear: the new phenomenon is analogous to the SdH effect. Therefore at large values of $n(H)$ the increase of the field and the decrease of the number of levels should lead to an increase of the amplitude. This increase should be enhanced because of the dependence of the specularity coefficient on the angle at which the electrons arrive at the surface $q(\phi)$, inasmuch as the angle ϕ decreases with increasing field. However, as H approaches H_d, the relative number of electrons that are subject to redistribution over the levels decreases, and this leads to a decrease of the amplitude. In fields $H > H_d$, the decrease of the amplitude of the oscillations with decreasing field is caused (at $\omega\tau \gg 1$) primarily by the absolute decrease of the number of electrons that do not collide with the surface.

Finally, we note the undisputed fact that observation of this phenomenon offers evidence of the large magnitude of the specularity coefficient even for incidence angles close to 90°. Thus $\phi_1 \approx 55°$ for $Sb-1$ at H_1, and $\phi_1 \approx 75°$ for $Sb-2$.

One can expect new possibilities for the measurement of the linear dimensions of the Fermi surfaces of metals and for the study of the dependence of the specularity coefficient on the arrival angle.

[1]The unusual behavior of the monotonic part of the resistance $R(H)$ and accordingly of $\partial R/\partial H(H)$ can be explained within the framework of the static size effect and will not be considered here.

[2]The square brackets denote that it is necessary to take the integer part of the expression.

[3]For H_2, just as for H_1 in the case of the thinnest samples, it is necessary to take into account terms of fifth order in H (this was done for $Sb-3$, $d=0.08$ μ), and the ellipticity of the cross section.

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