Multiple processes in proton production from nuclei in a region kinematically forbidden to *NN* interaction

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The character of the dependence of the cross section for the production of protons from a nucleus A by high-energy particles on the emission angle θ $(1 \leq \theta < \pi)$ and on the proton momentum k ($k \gtrsim 2$–3 GeV/c) on account of multiple rescatterings by the nucleons of the nucleus is explained. It is shown that rescattering plays an insignificant role in the region $k > 2Am_N/\theta^2$.

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The laws governing the production of protons and mesons from nuclei in a region that is kinematically forbidden to *NN* interaction has recently attracted ever increasing attention in connection with the hope of obtaining new information on nuclear structure.[1–5] At the same time, the emitted particle can land in the forbidden region as a result of rescattering by individaul nucleons of the nucleus, i.e., as a result of a process in which large distances between nucleons play an important role. It was shown in[6], for the case of pion production from a deutron, that rescattering of pions (as well as of a proton in the initial state) is significant at $k_{\pi\perp} \neq 0$.

In this paper we consider the production of protons from heavy nuclei in the region where large interaction multiplicities are important, $n > \theta^2$, where θ is the angle of the proton emission ($1 \lesssim \theta < \pi$). As will be shown later on, this condition means $k \gtrsim 2$–3 GeV/c. The limitation on the nuclear dimension, since the nucleon must experience a sufficient number of rescatterings, takes the form $R > l\sqrt{n}/\theta$, where l is the mean free path in nuclear matter, i.e., $A > (4r_0^2 n/\theta\sigma_{pp}^{tot})^3$, and $R = A^{1/3}r_0$. For $\theta = \pi$ this condition starts to be satisfied at $A \sim 100$. At smaller A it is necessary to take into account geometric factors that lead to a decrease of the rescattering probability. The considered process proceeds in the following manner: the incident particle (for the sake of argument, a proton) of high energy ($E \gg k$) produces from one of the nucleons of the nucleus a nucleon of finite energy that depends only on k and θ, and this nucleon then experiences consecutive rescatterings by other nucleons of the nucleus. It can be shown that the maximum momentum in the θ direction after n rescatterings by immobile nucleons is

$$k_n^{max} = \frac{2m\zeta\cos^n(\theta/n)}{1-\zeta^2\cos^{2n}(\theta/n)} \tag{1}$$

and is reached if the scattering in each act is through an angle $\sim\theta/n$. ($\zeta^2 = (E-m)/(E+m)$ and m is the mass of the nucleon.) Approximately we have $k_n^{max}(\theta) \approx 2 \approx n/\theta^2$, which exceeds the exact value of k_n^{max} by only 30% even at $n = 4$ and $\theta = \pi$. It follows from this expression that the contributions from the processes with rescattering have a characteristic dip at $\theta \sim \pi$, and at $k > k_A = 2mA/\theta^2$ the rescatterings are significant only when account is taken of the motion of the nucleons in the nucleus. Therefore measurement of the cross section in this region would indeed yield nontrivial information on the structure of the nucleus. At $E - 6$ GeV we have $k_A = 1.6$ GeV/c and 1 GeV/c for C^{12} and Li^7, respectively. At not too large k ($k \lesssim 5$ GeV/c), the dissociation processes are of little significance in all the interaction acts except the first.

Calculation of the contribution of the process of n-th multiplicity to the function $f = \omega(d^2\sigma/d^3k)$ will be carried out quasi-classically, i.e., assuming that it contains the products of the probabilities of the interactions with each of the nucleons. If n is large enough, the calculations can be greatly simplified by taking out the value of the cross section $(d\sigma/dt)_i$ at the point (s_i, t_i) corresponding to scattering through an angle θ/n : $s_i = 4m^2n^2/\theta^2(i-1)$, $t_i = 4m^2n^2/\theta^2 i(i-1)$, $1 < i \leq n$. The calculation then reduces to the determination of the phase volume

$$\Phi \sim \frac{1}{\sqrt{n}(n-2)!}\left[\frac{\pi}{\sqrt{2}}\frac{\theta^2}{n^2}(n-n_0)\right]^{n-2}, \quad n_0 = \frac{k\theta^2}{2m}.$$

The answer for the contribution of the process of n-th multiplicity f_n depends on the parametrization of the scattering cross section. We choose the parametrization in the form

$$\left.\frac{d\sigma}{dt}\right|_{pp} = \frac{C_0}{s^\alpha t^\beta}, \quad \left.\omega\frac{d^3\sigma}{d^3k}\right|_{pp\to p\ldots} = C_1\frac{e^{-bk_\perp}}{1-x},$$

where, according to[8], $\alpha \approx 7$ and $\beta \approx 5$. Then, at $n \sim 5$–15, we can represent f_n in the following form, which is convenient for subsequent analysis:

$$f_n \approx \pi R^2 C B^{n-1}(\theta) n^D (n-n_0)^{n-2} \frac{\ln E/m}{n^{(\alpha+1)n}}, \tag{2}$$

where $\ln B \approx 12.5$ at $\theta = \pi$ and ~ -8 at $\theta = \pi/2$; $D = 3\alpha/2 + 2\beta + 1$; C is expressed in terms of C_1 and b:

$$C = \frac{C_1}{\sigma_{pp}^{tot}} \exp\left[-a\frac{\theta^2}{4} - \frac{2mb}{\theta}\right](2\pi)^{\frac{a-3}{2}+\beta}.$$

Expression (2) can be easily summed over n by writing $n = n_0(1+\lambda)$ and taking λ to be a small quantity. In this case

$$f_n \approx \pi R^2 C \frac{B^{n_o-1} n_o^{D-2}}{n_o^{an_o}} e^{-\phi\lambda} \lambda^{n_o-2} \ln\frac{E}{m}, \tag{3}$$

$$\phi = n_o[a(\ln n_o + 1) - \ln B + 1] - D.$$

The conditions under which f_n can be represented in the form (3) are $\lambda^2 D/2 < 1$ and $\lambda^2 \alpha n_0/2 < 1$. When these conditions are no longer satisfied, the analysis is only qualitative in character.

Expression (3) has a maximum at $\lambda^{max} = (n_0 = 2)/\phi$. With increasing k we have $\lambda_{max} \to [\alpha(\ln n_0 + 1)]^{-1}$, i.e., the optimal multiplicity of the interaction, $n = n_0(1 + \lambda^{max})$, differs little from $n_0 = k\theta^2/2m$—which is required by kinematics. λ^{max} increases small n, when $\alpha n_0 \ln n_0 \sim D$. This means (particularly when the Fermi motion is taken into account, see below), that at smaller n it is necessary to take into account interaction multiplicities that are significantly larger than that needed kinematically. Equation (3) can be easily integrated with respect to n, and we obtain

$$f \approx \pi R^2 C \frac{B^{n_o-1} n_o^{D-1} \Gamma(n_o - 1)}{n_o^{a n_o} \phi^{n_o-1}} \ln\frac{E}{m}, \tag{4}$$

where Γ is the Gamma function. At $\alpha n_0(\ln n_0 + 1) \gg D$ we have

$$f \approx \pi R^2 C \frac{(B/ea)^{n_o-1} n_o^{(D-0,5)}}{n_o^{an_o} (\ln n_o + 1)^{n_o-1}} \ln\frac{E}{m}. \tag{5}$$

We shall take the Fermi motion of the nucleons into account by assuming that the nucleon momentum distribution has a sharp boundary: $\rho(\mathbf{p}) = (3/4\pi p_F^2)\theta(p_F - p)$. The influence of the Fermi motion causes the kinematic boundaries to broaden in the following manner

$$k_n^{max}(\theta) = \frac{2mn}{\theta^2}(1+\gamma), \quad \gamma = \frac{p_F}{2m}\left(\theta + \frac{1}{\theta}\right), \quad n_o \to n_F = \frac{k\theta^2}{2m(1+\gamma)}$$

and this boundary is reached only if each of the nucleons from which the scattering took place had a momentum $p = p_F$ in a definite direction. We can obtain a simple answer for the function f_n with allowance for the Fermi motion at a sufficiently large multiplicity n. In expression (4) it is necessary to make the substitutions

$$\approx B\frac{(m\theta)^2(1+\gamma)^3 e^{3/\theta}}{2(3p_F)^2},\ C \to C_F,\ \phi \to \phi_F = n_F[a(\ln n_F + 1) - \ln B_F + 3] - D,$$

$$\Gamma(n_0 - 1) \to \Gamma(3n_F - 1),\quad \phi^{n_0 - 1} \to \phi_F^{3n_F - 1}.$$

Accordingly, $\lambda_F^{max} = (3n_F - 2)/\phi_F$, i.e., when the Fermi motion is taken into account, the optimal multiplicity becomes larger than n_F. This result is meaningful, of course, at $1+\lambda_F^{max} \lesssim (1+\gamma)(1+\lambda^{max})$, inasmuch as allowance for the Fermi motion makes smaller values of n significant at given k and θ.

Thus, the most rapidly varying factors in the cross section for proton production from nuclei are of the form

$$\left[B'(\theta)\left(\frac{2m(1+\gamma)}{k\theta^2}\right)^a\right]^{k\theta^2/2m(1+\gamma)}\left(\frac{k\theta^2}{2m}\right)^D,$$

i.e., they are determined by the behavior of the cross section for elastic p^N scattering.

For the effective slope $\Lambda = -\partial \ln f/\partial k^2$, we obtain the expression

$$\Lambda \approx \frac{\theta^2}{4mk}\left[a\left(\ln\frac{k\theta^2}{2m} + 1\right) - \ln\frac{B(\theta)}{ea}\right] - \frac{D}{2k^2}, \qquad (6)$$

which leads at $k^2 = 1$ (GeV/c)^2 (outside of the applicability region!) to values of the required scale. Expression (6) can be compared with experiment at least at $k \gtrsim 2$ GeV/c, and with allowance for geometrical factors of the form

$$\int^{\theta R/l_0} e^{-l}\frac{l^{n-1}dl}{(n-1)!}$$

which are important for light nuclei and which leads to a stronger dependence on A than $f \sim A^{2/3}$.

It was shown in[7] that at small momentum, $k \lesssim 0.5$ GeV/c, a twofold interaction makes a contribution smaller by several times than the experimental cross section. The conclusion that it is necessary to take into account interactions with $n \gtrsim 3$—4 even at small values of k is confirmed by the fact that the value of λ increases with decreasing k.

The method developed here makes it possible to calculate numerically f at $n < k\theta^2/2m(1+\gamma)$, when the simple expression (5) does not hold. The lower bound of n is connected with the fact that at small n it is impossible to take the scattering cross section outside the integral sign at a fixed point (s_i, t_i).

The obtained rules are valid for arbitrary particles in a beam and in the final state, all that change are certain details of the kinematics of the process.

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[1]Yu. D. Bayukov *et al.*, Izv. Dokl. Akad. Nauk SSSR Ser. Fiz. **30**, 521 (1966); Pis'ma Zh. Eksp. Teor. Fiz. **21**, 461 (1975) [JETP Lett. **21**, 210 (1975)]; Yad. Fiz. **18**, 1246 (1973) [Sov. J. Nucl. Phys. **18**, 639 (1974)].
[2]A. M. Baldin *et al.*, Yad. Fiz. **18**, 79 (1973); **18**, 1246 (1973); **21**, 461 (1975) [Sov. J. Nucl. Phys. **18**, 41 (1974); **18**, 639 (1974); **21**, 657 (1975)].
[3]J. Papp *et al.*, Phys. Rev. Lett. **34**, 601 (1975).
[4]G. A. Leksin, Yadernyĭ skeĭling (Nuclear Scaling), Lectures of Moscow Eng. Phys. Inst., 1975.
[5]G. A. Lobov, V. E. Markushin, V. V. Solov'ev, and I. S. Shapiro, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 118 (1976) [JETP Lett. **23**, (1976)]; L. L. Frankfurt and M. I. Strikman, Leningrad Inst. Nuc. Phys. Preprint No. 173, 1975.
[6]L. A. Kondratyuk and V. B. Kopeliovich, Pis'ma Zh. Eksp. Teor. Fiz. **21**, 88 (1975) [JETP Lett. **21**, 40 (1975)].
[7]V. B. Kopeliovich, Paper at 4th Internat. Seminar on High-Energy Physics, 5-11 June 1975.
[8]R. Blankenbecler, Lect. Notes Physics **32**, 1 (1975).