Leptonic decays of heavy pseudoscalar mesons

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It is indicated that $SU(4)$ symmetry breaking due to the increased mass of the s and c quarks can lead to an appreciable fraction of two-particle lepton decays of heavy (charmed) pseudoscalar mesons. The difference between the experimental values of the vector and axial Cabibbo factors can be explained within the framework of the developed model.

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1. The experimental data indicate that the polarization of the direct μ^+ mesons produced with large transverse momenta in nucleon-nucleon collisions is predominantly the same as in two-particle decays of π and K mesons.[1] If the source of the direct leptons are decays of (charmed) particles (and this assumption seems to be favored by the increase of the relative μ/π yield with increasing energy), this raises, in the usual weak-interactions with left-hand lepton currents, the question of what is the fraction of two-particle lepton decays of heavy pseudoscalar mesons.

It is usually stated that the relative probability of such decays is low. The starting point is the matrix element $M=(G_C/\sqrt{2})f_P k_\alpha l_\alpha \phi_P$ (where k_α and ϕ_P are the momentum and the wave function of the meson P, while l_α is the lepton current, $G_C = G\cos\theta_C$ or $G_C = G\sin\theta_C$, θ_C is the Cabibbo angle, and f_P is a constant), which leads to a $P\to l\nu$ decay probability proportional to the pseudoscalar meson mass M_P and the square of the lepton mass μ:

$$W_{P\to l\nu} = \frac{G_C^2}{8\pi} f_P^2 M_P \mu^2 \left(1 - \frac{\mu^2}{M_P^2}\right)^2 .$$

At the same time, the total probability of the semileptonic decays (which is assumed in the quark model to be equal to the probability of the c-quark decay $c\to s(u,d)+l+\nu$) turns out to be proportional to the fifth power of the mass of the heavy quark

$$W = \frac{G_C^2 m_c^5}{192\pi^3} F(x) \quad , \qquad x = \frac{m^2_{s(u,d)}}{m_c^2}, \tag{1}$$

$$F(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x. \tag{2}$$

Therefore, when it is assumed that in the $SU(4)$ symmetry approximation $f_P = f_\pi = 0.96 m_\pi$, the value obtained for the probability of the $P \to \mu\nu$ decays (without suppression by the Cabibbo factor) a value amounting to $1-2\%$ of the total probability of the semileptonic decays.

In this article, on the basis of the quark model, we advance arguments that the constant f_P is indeed proportional to the sum of the masses of the quarks making up to meson.[1)] That is to say, the constant f_P can change strongly when the symmetry breaking is due to the increased mass of the quark.

Indeed, if it is assumed that the coupling of the meson with its constituent quarks is of the form $g_P(\bar{q}_1\gamma_5 q_2)\phi_P$, where g_P is a universal constant, then the consideration of the $P \to \mu\nu$ decay via the $(q_1\bar{q}_2)$ loop yields for f_P the expression

$$f_P = \frac{g_P(m_1 + m_2)}{8\pi^2}\left[\ln\frac{\Lambda^2}{m_1 m_2} + \Delta(m_1, m_2, m_P)\right], \tag{3}$$

where Λ is the cutoff constant, m_1 and m_2 are the quark masses, and $\Delta(m_1, m_2, m_P)$ is a definite function of the masses m_1, m_2, and m_P. In the subsequent estimates we shall use for the quark masses the values

$$m_u = m_d \approx 350\ \text{MeV},\ m_s \approx 500\ \text{MeV}, \quad m_e \approx 1500\ \text{MeV}.$$

Taking as the constant for the pseudoscalar coupling of the mesons with the quarks $g_P/\sqrt{2} = 2m_d f_q/m_\pi$, where $f_q = \frac{3}{5}f$ and f is the constant of the pseudovector πNN coupling, $f^2/4\pi = 0.082$ (see[2]), we can use the experimental value of f_π to estimate from (3) the value of cutoff constant $\Lambda \approx 3.5$ GeV.

Comparison of the probabilities of the $\pi \to \mu\nu$ and $K \to \mu\nu$ decays does not contradict the indicated dependence of f_P on the quark mass. Moreover, it becomes possible to understand the discrepancy between the Cabibbo axial and vector factors. Indeed, according to (3), in the limit $\Lambda^2 \gg m_1 m_2$ the ratio is

$$r_A = \frac{\operatorname{tg}\theta_A}{\operatorname{tg}\theta_C} = \frac{f_K}{f_\pi} = \frac{m_s + m_u}{m_u + m_d} = 1.2 ,$$

whereas the experimental values ($\sin\theta_C = \sin\theta_V = 0.208 \pm 0.007$ and $\sin\theta_A = 0.264 \pm 0.001$)[3] yield a value $r_A = 1.28 \pm 0.04$.

Applying the same considerations to the decay of the meson $F(c,s)$, which proceeds according to[4] without suppression of the Cabibbo factor, we have

$$\frac{W_{F\to\mu\nu}}{W_{\pi\to\mu\nu}} = \frac{m_F}{m_\pi}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^{-2}\left(\frac{m_c + m_s}{m_u + m_d}\right)^2 .$$

For $m_F = 1.9$ GeV, $W_{F\to\mu\nu} \approx 620$ and $W_{\pi\to\mu\nu} \approx 2.4\times10^{10}$ sec^{-1}. On the other hand, the total probability of the semileptonic decays $F\to\mu\nu X$ amounts, according to the upper-bound estimate (1), to $W_{F\to\mu\nu X}\approx 1.2\times10^{11}$ sec^{-1}. Thus, if the indicated considerations are valid, the probability of the two-particle $F\to\mu\nu X$ decays is about 20% of the probability of all the semileptonic $F\to\mu\nu X$ decays.[2)]

In the standard scheme,[4] the arguments advanced that the fraction of two-particle leptonic decays is large are valid only for $F\to\mu\nu$, since the $D\to\mu\nu$ decays are suppressed by the Cabibbo factor. If, on the other hand, a hadronic current $(\bar{c}d)$ exists, having a $(V+A)$ structure without suppression, then the conclusions concerning the appreciable role of the two-particle decays remain in force also for D mesons (as well as for the case of schemes with several heavy quarks).

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[1)]The statement that $f_P \sim (m_1+m_2)$ can also be obtained from PCAC considerations for the divergence of the current $\bar{q}_1\gamma_\mu\gamma_5 q_2$ with allowance for the Klein-Gordon equation as $k^2\to 0$, for a field ϕ_P with density $\bar{q}_1\gamma_5 q_2$ as a source.
[2)]Estimates show that for $m_c = 1.5$ GeV and $m_F = 1.9$ GeV the semileptonic modes are practically restricted to the channels $F\to l\tilde{\nu}_e\eta$, $l\tilde{\nu}_e\phi$, and $l\tilde{\nu}_e\eta'$, where l is an electron or muon (in 50% of the cases there are no charged hadrons among the decay products).

[1]B.A. Dolgoshein, Yu.P. Nikitin, and G.V. Rozhnov, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 381 (1975) [JETP Lett. **22**, (1975)].
[2]J. Kokkedee, Quark Model, Benjamin, 1969.
[3]J. Sakurai, Currents and Mesons, U. of Chicago, 1969. E.I. Mal'tsev and I.V. Chuvilo, Fiz. Elem. Chastits At. Yadra **1**, 443 (1971) [Sov. J. Part. Nucl. **1**, 105]
[4]S.L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. **D2**, 1285 (1970).
[5]M. Ruderman and R. Finkelstein, Phys. Rev. **76**, 1458 (1949).