Spatial coherence produced in laser radiation on passing through the generation threshold

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The behavior of the spatial statistics of a laser field when the generation threshold is approached is investigated. It is established that the correlation radius $R_{cor} \sim (\delta - \delta_{thr})^{-1/2}$, where δ_{cor} is the loss at which lasing sets in. The data obtain‹ can be used to identify the laser regime. Attention is called to the fact that the experimentally obtained relations $R_{cor} = r_{cor}(\delta)$ is in good agreement with the Landau self-consistent-field theory for second-order phase transitions. (The critical exponent is very close to the theoretical value 0.5; experiment yields 0.49 ± 0.05.)

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1. We report here the results of an experimental and theoretical study of th‹ formation of spatial coherence in lasers with feedback. The law governing the variation of the transverse-correlation radius r_{cor} as the lasing threshold is a| proached is traced here for the first time. It is shown that the behavior of the correlation radius near the lasing threshold agrees well with the self-consiste field theory for second-order phase transitions.

2. Spatial coherence (SC) is one of the most important characteristics of laser radiation. The question of the ultimate SC of laser radiation operating above threshold can by now be regarded as answered (see[1]). Of considerable interest is the investigation of the formation of SC when the lasing threshold is approached. It is precisely in this manner that it is possible to trace quant: tatively the transition from the stochastic field of the spontaneous emission to the lasing regime. Such investigations are of interest also from two other poi of view. Knowledge of the regularities of the spatial statistics can be used to identify the laser action; it is known that in a number of cases (particularly fc short-wave lasers) this is far from a trivial problem.

The other interesting aspect is connected with some general properties of lasing. Attention was called in a number of theoretical papers (see[2]) to the analogy between self-excitation of a laser and second-order phase transitions From this point of view, measurements of the SC of a laser field on passing through threshold are analogous to measurements of the spatial correlations ‹ critical points. The results described below offer evidence that this analogy i not merely qualitative in character.

3. In experiment, using a polarized interferometer,[3] we measured the transverse correlation function $\Gamma(s) = \langle E(r,t) E(r+s,t) \rangle$ of the radiation field $E(r,t)$ of a cw gas laser at different values of the loss introduced in to the resonator.

The high sensitivity (photomultiplier dark current ~ 3 pulses/sec) and the high spatial resolution (we could measure light-field correlation radii $\sim 10 \pm 2$

of the measurement setup made possible careful measurements of the SC ›t only above but also below the threshold.

The measurements were performed with an LG-159 He-Ne laser ($\lambda = 0.63\ \mu$) ith an absorbing neon cell.

The operating regime was varied by taking the resonator out of adjustment y changing the losses in the resonator) at a given tube current and a certain ›rking current I_w of the neon cell. Lasing set in when I_w was decreased to $_{\rm tr}$, the radiation intensity increasing jumpwise by more than four orders of agnitude. The decrease of the current I_w was accompanied by a decrease of e losses introduced by the cell, owing to the decrease in the population of the wer ($2p^5 3p$) of the Ne levels, to which a transition to $\lambda = 0.63\ \mu$ corresponds. the measurements we took into account the hysteresis on the power curves.[4]

The figure shows the dependence of the field transverse-correlation radius the parameter $\eta = (I_w - I_{\rm thr})/I_w$ that characterizes the degree of approach to e lasing threshold.

The slope of the curve in Fig. b equals -0.49, i.e., the relation $r_{\rm cor} \sim \eta^{-\alpha}$ satisfied, where $\alpha = 0.49 \pm 0.05$ (with allowance for the measurement error.

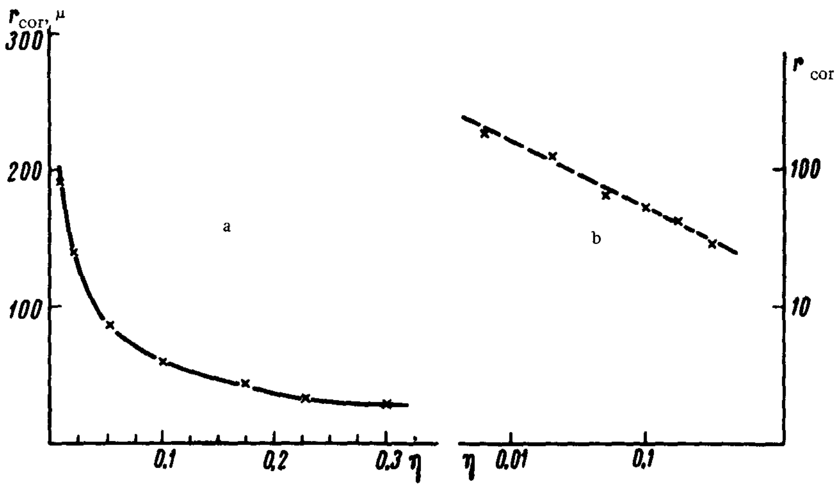

FIG. Dependence of the transverse-correlation radius of the laser field on the loss parameter $\eta = (I_w - I_{thr})/I_w$: a--linear scale, b--logarithmic scale.

4. The theoretical dependence of the radius $r_{\rm cor}$ on the lasing conditions can obtained by resorting to the frequency-angle distribution of the spontaneous ıission at the exit of a Fabry-Perot interferometer filled with an active ›dium[5]

$$S(\nu, \theta) = (1 - r_2^2)(G^2 - 1)(\gamma - \delta)^{-1}[(1 - r_1 r_2 G^2)^2 + 4 r_1 r_2 G^2 \sin^2\Phi]^{-1} S_0(\nu, \theta), \qquad (1)$$

ere $S_0(\nu, \theta)$ is the corresponding distribution for the spontaneous emission,[6] ; the frequency, θ is the polar angle reckoned from the normal to the mir- ', r_i is the mirror amplitude reflection coefficient, $G = \exp[(\gamma - \delta)l/2]$ is the amplitude gain, γ is the growth rate, δ is the decrement, l is the resonator length, $\Phi = kl\cos\theta$ is the phase shift, and k is the wave number.

The transverse spatial coherence function of the radiation at the exit from the resonator is given by

$$\Gamma(\nu,s) = 2\pi k^2 \int_0^{\pi/2} J_0(ks\sin\theta)\, S(\nu,\theta)\sin\theta\, d\theta\,, \quad (2)$$

where $J_0(x)$ is a Bessel function.

In the approximation in which the divergence angle is small we obtain for the normalized SC function from (1) and (2)

$$|\gamma(s)| = |\Gamma(\nu,s)| \Big/ |\Gamma(\nu,0)| = |\mathrm{kei}\,(\beta ks)| \Big/ \mathrm{kei}\,(0), \quad (3)$$

where kei(x) is a Thomson function and $\beta^2 = (1 - r_1 r_2 G^2)/[(r_1 r_2)^{1/2} Gkl]$. Near the laser excitation the correlation radius is

$$r_{\mathrm{cor}} = (2l/k)^{1/2} (r_1 r_2 G^2)^{1/4} (1 - r_1 r_2 G^2)^{-1/2} \quad . \quad (4)$$

This expression is transformed into

$$r_{\mathrm{cor}} \approx (2GG_{\mathrm{thr}}/k)^{1/2} (\delta - \delta_{\mathrm{thr}})^{-1/2} \quad (5)$$

by introducing the threshold value $G_{\mathrm{thr}} = (r_1 r_2)^{-1/2}$ and expanding $G^2 = 1 + (\gamma - \delta)l$ at low values of the gain and under the condition that when the lasing threshold is approached the losses in the resonator change. We write down the dependence of the losses on the current in the form $\delta = f(I_w)$, and then $\delta - \delta_{\mathrm{thr}} = f'(I_{\mathrm{thr}})(I_w - I_{\mathrm{thr}})$, $r_{\mathrm{cor}} \sim \eta^{-1/2}$, and $\alpha = 0.5$.

5. The results of the calculation and of the experiment are in agreement. The law $r_{\mathrm{cor}} \sim (\delta - \delta_{\mathrm{thr}})^{-1/2}$ describes well the behavior of a field with feedback near the lasing threshold. We note that a direct calculation of the correlation radius by formulas (4) and (5) is difficult, since it is necessary to know, besides the mirror losses, also the diffraction losses and the losses introduced by the absorbing cell. It is possible, however, to connect the radiated power P with the introduced losses and obtain the dependence of r_{cor} on P[5]; the value $r_{\mathrm{cor}} = 100\ \mu$ corresponds to a radiated power 1×10^{-15} W.

6. The relation obtained for the correlation radius is also in splendid agreement with the results of the self-consistent-field theory of second-order phase transitions.[7] (The pump parameter or the loss parameter δ is in this case the analog of the temperature—see[2].) It is important to emphasize that the critical exponent agrees much better with calculation by the self-consistent-field theory for a laser than for other physical systems (cf.[8]).

The foregoing makes desirable a theoretical analysis of lasing, in greater detail than in[2], from the point of view of nonequilibrium phase transitions. The laser may turn out to be a convenient physical system for the study of critical phenomena. Furthermore, this approach can yield general results that are of interest for laser physics. Thus, interest attaches to calculation of fluctuations and higher correlations near the lasing threshold; the existing experimental technique makes it possible to perform the corresponding measurements.

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