Microscopic theory of the stationary state of a parametrically-excited magnon system

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A system of kinetic equations, describing the behavior of magnons under parametric excitation in a temperature region greatly exceeding the energy of the parametrically-excited magnons, is obtained on the basis of a microscopic theory.

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A theory of parametric excitation of magnons was developed recently within the framework of the generalized self-consistent field (see the bibliography in[1].

Zakharov, L'vov, and Starobinets,[1] in their calculation of the steady-state distribution of the magnons, established the relaxation phenomenologically. The form of the collision integral corresponded to small deviations of the magnon distribution function from equilibrium. This choice of the collision integral, which does not take into account the influence of pumping and the self-consistency effect, leads to a distribution significantly different from equilibrium in the parametric-excitation region $\omega_k \approx \omega/2$ (ω is the frequency of the extremal field). A consistent kinetic analysis, generally speaking calls for taking the self-consistency into account in the collision integral. Such an analysis was carried out by Tsukernik and one of us,[2] who used an interaction Hamiltonian that conserves the number of excitations. The steady-state distribution obtained in[2] caused the separate vanishing of the convective part of the kinetic equation and of the collision integral. This has made it possible to get around the main difficulty connected with finding the explicit form of the collision integral. However, such a solution method is not applicable when the Hamiltonian contains terms that do not conserve the number of particles, for example cubic anharmonicities that make a significant contribution to the relaxation of magnons with small **k**.

In the present paper, when deriving the system of kinetic equations describing the behavior of a ferromagnet under conditions of parametric excitation by a uniform high-frequency magnetic field oriented along the easy axis, we take into account also processes which do not conserve the number of magnons. The field is assumed to be weak enough to make the characteristic energy of the interaction of the spin waves of the field μh_0 (μ is the Bohr magneton) small in comparison with the gap ω_0 in the spin-wave spectrum: $\mu h_0 \ll \omega_0$. It suffices therefore to take into account in the Hamiltonian of the ferromagnet only the resonant interaction with the external field, which leads in the linear theory to an exponential growth of the occupation numbers of the spin waves in the region of the parametric excitation[3]

$$H = \sum_{\mathbf{k}} \left[\omega_{\mathbf{k}} a_{\mathbf{k}}^{+} a_{\mathbf{k}} + \frac{1}{2}(V_{\mathbf{k}} a_{\mathbf{k}} a_{-\mathbf{k}} e^{i\omega t} + V_{\mathbf{k}}^{*} a_{\mathbf{k}}^{+} a_{-\mathbf{k}}^{+} e^{-i\omega t})\right] + H_{int} \tag{1}$$

$$\omega_{\mathbf{k}} = \sqrt{A_{\mathbf{k}}^2 - |B_{\mathbf{k}}|^2}, \qquad V_{\mathbf{k}} = \mu h_0 B_{\mathbf{k}}/2\epsilon_{\mathbf{k}}.$$

The magnon-interaction Hamiltonian

$$H_{int} = \frac{1}{N}\sum_{1234} \Phi_{12.34} a_1^+ a_2^+ a_3 a_4 + \frac{1}{N}\sum_{1234} (\phi_{1.234} a_1^+ a_2 a_3 a_4 + \text{H.c.}) + \frac{1}{\sqrt{N}}\sum_{123} (\phi_{1.23} a_1^+ a_2 a_3 + \text{H.c.}) \quad (2)$$

describes processes of exchange and relativistic origin, which conserve the number of excitations, as well as purely relativistic interactions, for which the number of particles is not conserved. The first group of processes is taken partially into account within the framework of the self-consistent field, and this leads to a renormalization of the spectrum and of the pump[1]:

$$\tilde{\omega}_{\mathbf{k}} = \omega_{\mathbf{k}} + \sum_{\mathbf{k}'} T_{\mathbf{k}\mathbf{k}'} n_{\mathbf{k}'}, \qquad P_{\mathbf{k}} = V_{\mathbf{k}} + \sum_{\mathbf{k}'} S_{\mathbf{k}\mathbf{k}'} \sigma_{\mathbf{k}'}$$

$$n_{\mathbf{k}} = \langle a_{\mathbf{k}}^+ a_{\mathbf{k}} \rangle, \qquad \sigma_{\mathbf{k}} = \langle a_{\mathbf{k}} a_{-\mathbf{k}} \rangle . \quad (3)$$

The remaining interactions are taken into account to the extent to which they lead to relaxation.

We consider next the case of a sufficiently high temperature $\Theta \gg \omega_0$. In addition, the pump amplitude is assumed to be small enough to make the number of parametric excitations of the magnons significantly smaller than the number of the thermal excitations, $N_P \ll N_\Theta$. These conditions together with the inequality $\mu h_0 \ll \omega_0$ indicated above are the main premises of the theory and lead to a physically lucid picture of the phenomenon.

By virtue of the inequalities $\Theta \gg \omega \gg \mu h_0$ we can separate a narrow interval of frequencies $\omega_0 < \omega_k < \Delta$, which includes in the linear theory a parametric-excitation region of width Δ, in which the magnon distribution function is essentially not in equilibrium. The magnons of this region of the spectrum will be called parametric, and the remainder will be called thermal; their creation and annihilation operators will be designated a_k^+, a_k and b_k^+, b_k, respectively. That part of the interaction Hamiltonian which describes the magnon collisions can be naturally broken up now into three terms corresponding to the separate collisions of the parametric magnons (H_P), the thermal magnons (H_Θ), and their interactions with one another ($H_{P\Theta}$). The principal role in the term H_Θ is played by the exchange interaction, which establishes a quasi-equilibrium Bose distribution in the thermal-magnon subsystem. The relaxation of the long-wave parametric magnons is, in the main, of relativistic origin, since the exchange-interaction amplitude is small when at least one of the wave vectors of the colliding magnons is small. Since $N_P \ll N_\Theta$, the main contribution to this relaxation is made by that part of $H_{P\Theta}$ which is linear in the operators a_k^+ and a_k:

$$H'_{P\Theta} = \sum_{\mathbf{k}} (J_{\mathbf{k}} a_{\mathbf{k}}^+ + J_{\mathbf{k}}^+ a_{\mathbf{k}}) . \quad (4)$$

It is precisely the Hamiltonian (4) which determines the form of the collision integral for the parametric magnons, and it exerts an appreciable influence on the formation of the quasi-equilibrium distribution of the thermal magnons.

The operator coefficients J_k can be naturally represented as sums of partial terms

$$J_{\mathbf{k}} = \sum_l J_{\mathbf{k}l}$$

where $l = 0, \pm 1, \pm 2$ are the differences between the number of the annihilation operators b_k and creation operators b_k^+ of the thermal magnons in different terms of the Hamiltonian $H'_{P\Theta}$. In accordance with (4), the dissipative parts of the kinetic equations for n_k and σ_k are of the form

$$i\left(\frac{dn_{\mathbf{k}}}{dt}\right)_{\text{coll}} = \langle J_{\mathbf{k}} a_{\mathbf{k}}^+ \rangle - \langle J_{\mathbf{k}}^+ a_{\mathbf{k}} \rangle ,$$
$$i\left(\frac{d\sigma_{\mathbf{k}}}{dt}\right)_{\text{coll}} = \langle J_{\mathbf{k}} a_{-\mathbf{k}} \rangle + \langle J_{-\mathbf{k}} a_{\mathbf{k}} \rangle . \tag{5}$$

The averaging in these equations is carried out with the density matrix $\rho = \rho_0 + \rho_1$. Here $\rho_0 = \rho(b)\rho(a)$, $\rho(b)$ is a Gibbs distribution with an indeterminate chemical potential ξ, $\rho(a)$ is the non-equilibrium density matrix of the parametric magnons, and ρ_1 is an increment of first order in $H'_{P\Theta}$ to ρ_0. It turns out as a result that the collision integrals (5) are expressed in terms of partial damping of the single-particle excitations

$$\gamma_{\mathbf{k}l} = \left(\int_{-\infty}^{\infty} \langle [J_{\mathbf{k}l}(t), J_{\mathbf{k}l}^+(0)] \rangle e^{iEt} dt \right)_{E = \omega_{\mathbf{k}}} \tag{6}$$

Eliminating the "rapid" time dependence from σ_k and P_k ($\sigma_k \to \sigma_k \exp(-i\omega t)$, $P_k \to P_k \exp(-i\omega t)$), we obtain finally kinetic equations for the parametric magnons

$$i\dot{n}_{\mathbf{k}} = P_{\mathbf{k}}\sigma_{\mathbf{k}}^* - P_{\mathbf{k}}^*\sigma_{\mathbf{k}} + 2i \sum_l \frac{\gamma_{\mathbf{k}l}}{\tilde{\omega}_{\mathbf{k}} - \xi l} \{ \Theta - (\tilde{\omega}_{\mathbf{k}} - \xi l) n_{\mathbf{k}} - \operatorname{Re} P_{\mathbf{k}}\sigma_{\mathbf{k}}^* \} , \tag{7}$$

$$i\dot{\sigma}_{\mathbf{k}} = (2\tilde{\omega}_{\mathbf{k}} - \omega)\sigma_{\mathbf{k}} + 2P_{\mathbf{k}} n_{\mathbf{k}} - 2i \sum_l \frac{\gamma_{\mathbf{k}l}}{\tilde{\omega}_{\mathbf{k}} - \xi l} \{ (\tilde{\omega}_{\mathbf{k}} - \xi l)\sigma_{\mathbf{k}} + P_{\mathbf{k}} n_{\mathbf{k}} \} . \tag{8}$$

Taken together with equations for the balance of the number of thermal magnons

$$\frac{dN_\Theta}{d\xi}\dot{\xi} = - \sum_{\mathbf{k},l} \frac{l\gamma_{kl}}{\tilde{\omega}_{\mathbf{k}} - \xi l} \{ \Theta - (\tilde{\omega}_{\mathbf{k}} - \xi l) n_{\mathbf{k}} - \operatorname{Re} P_{\mathbf{k}}\sigma_{\mathbf{k}}^* \} + \lambda \xi \frac{dN_\Theta}{d\xi} \tag{9}$$

(λ is the relaxation rate of the chemical potential[3]), Eqs. (7) and (8) constitute the complete system of equations of the problem. The assumption $N_P \ll N_\Theta$ allows us to regard the spin-system temperature as equal to the lattice temperature.

In the derivation of the collision integral we did not take into account the influence of the damping; in addition, we used the smallness of the characteristic value of the parametric-magnon energy in comparison with the temperature, also the known relation[4] between the Fourier components of the commutator and anticommutator averaged over the Gibbs distribution $\rho(b)$

$$< \{J_{kl}, J_{kl}^{+}\} > / < [J_{kl}, J_{kl}^{+}] > = \mathrm{cth}\frac{E - \xi l}{2\Theta} \approx \frac{2\Theta}{E - \xi l} \quad .$$

The validity of the foregoing subdivision of magnons into parametric and thermal is justified by the fact that the deviation of the stationary distribution of the parametric magnons from quasi-equilibrium decreases rapidly with frequency, so that the result does not depend on the parameter Δ.

If the interaction Hamiltonian does not contain terms that do not conserve the particle number, then $l = 1$, $\lambda = 0$, $\xi = \omega/2$, and the stationary position of the system (7)—(9) coincides with the high-temperature limit of the results of[4]. If the cubic anharmonicity is significant, so that $\lambda \sim \gamma_k$, then the paramagnetic magnons do not exert a noticeable influence on the thermal ones ($\xi/\omega \ll 1$), and the dynamic and dissipative parts of the kinetic equation cannot vanish independently. We note that in this case allowance for the pumping (with simultaneous neglect of the damping) in the collision integral is meaningful only for the nonresonant case $|\tilde{\omega}_k - \omega/2| > |P_k|$, when the two compared quantities can greatly exceed γ_k. In the resonant case ($\omega_k \sim \omega/2$), the renormalized pumping is comparable with γ_k, and the relative difference between the dissipative terms (7) and (8) from the terms phenomenologically introduced in[1] is of the order of γ_k/ω_k. Thus, within the framework of the employed approximation, allowance for the pumping in the collision integral for the resonant case is an exaggeration of the accuracy.

The foregoing analysis pertains to the idealized case of a uniaxial ferromagnet. In the case of ferrites and antiferromagnets, exchange interaction can also lead to nonconservation of the number of magnons.

[1]V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Usp. Fiz. Nauk **114**, 609 (1974) [Sov. Phys. Usp. **17**, 896 (1975)].

[2]V. M. Tsukernik and R. P. Yankelevich, Zh. Eksp. Teor. Fiz. **68**, 2116 (1975) [Sov. Phys.-JETP **41**, 1059 (1975)].

[3]A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskiĭ, Spinovye volny (Spin Waves), Nauka, 1967.

[4]D. I. Zubarev, Neravnovesnaya statisticheskaya termodinamika (Nonequilibrium Statistical Mechanics), Nauka, 1971.