# Magnetocaloric effect in an alternating field

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Pumping of excitations in a superconducting film by a low-frequency field, resulting in cooling of the superconductor, is considered.

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This paper is devoted to the dependence of the number of excitations in a superconductor on the periodic variation of the gap in their spectrum. At low temperatures it is possible to choose the frequency of the gap-changing alternating magnetic field in such a way that the phonon system has time to adjust itself to the electronic system. In this case it is easy to obtain the solution of the kinetic equation. The possibility of cooling a superconductor by changing the number of excitations by this method is also considered.

Consider a thin superconducting film of thickness $d \ll \xi$ and $d \ll \delta$, where $\xi$ is the coherence length and $\delta$ is the depth of penetration of the magnetic field. Let a vector potential $\mathbf{A}+\mathbf{A}_1+\mathbf{A}_2\cos\omega t$ be applied to the film, with $\mathbf{A}$ parallel to the film and $|\mathbf{A}_1| \gg |\mathbf{A}_2|$. We confine ourselves to the case of a film strongly "contaminated" with impurities and to frequencies $\omega\Delta \ll 1$, where $\Delta$ is the superconductivity parameter. Then, according to[1,2], the gap in the perturbation spectrum is

$$\epsilon_0 = \epsilon_0^{(0)} - \left(\frac{\epsilon_0^{(0)}}{a_0}\right)^{1/3} a_\omega \cos\omega t \;, \tag{1}$$

where $\epsilon_0^{(0)}$ is the gap in the absence of a time-alternating field $\mathbf{A}_2$,

$$a_0 = \frac{2}{3}\tau\frac{e^2}{c^2}v_F^2 \langle \mathbf{A}_1^2 \rangle ; \qquad a_\omega = \frac{2}{3}\tau\frac{e^2}{c^2}v_F^2 \langle \mathbf{A}_1\mathbf{A}_2 \rangle$$

$\tau$ is the time between the collisions with the impurities, $v_F$ is the electron velocity on the Fermi surface, $e$ is the electron charge, and $c$ is the speed of light.

The symbol $\langle\ \rangle$ denotes averaging over the thickness of the film. Expression (1) was obtained in the case $\alpha_0 \ll \epsilon_0$, to which we confine ourselves here.

Since the film thickness is small in comparison with the characteristic quantities $\xi$ and $\delta$, and since $\mathbf{A}_1$ and $\mathbf{A}_2$ are constant along the film, the problem is homogeneous. At $\mathbf{A}_1 = 0$ the corresponding kinetic equation was derived by Éliashberg.[3] At $\mathbf{A}_1 \neq 0$, a change takes place in the density of the excitation spectrum and in the coherence factors of the kinetic equation. These quantities were obtained in accordance with[1]. The new density of states is

$$\rho_\epsilon = \rho_0\left(\frac{2}{3}\right)^{1/2}\left(\frac{\Delta^2}{a\epsilon_0}\right)^{1/6}\sqrt{\frac{\epsilon-\epsilon_0}{a}} \;, \tag{2}$$

and the factors $1 \pm \Delta^2/\epsilon\epsilon_1$ change by $1 \pm (\epsilon_0/\Delta)^{2/3}$ if $\epsilon - \epsilon_0 \ll \alpha$, where $\alpha = \alpha_0$

  

$\alpha_\omega \cos\omega t$, and $\rho_0$ is the density of the spectrum of the normal metal on the ermi surface. We consider the case of low temperatures $T \ll \alpha$, and only val-es $\epsilon - \epsilon_0 \sim T$ are of importance to us.

In the collision integral of the kinetic equation there are terms that describe ıe processes of absorption and emission of a phonon by an electron, and a erm describing the disintegration of a phonon into two excitations and the in-erse process.

The time of the former processes $\tau_1 = \Theta_D{}^2 \Delta^{1/2}/\lambda T^{7/2}$ is much smaller at $T \ll \Delta$ then the characteristic time of the latter processes

$$\tau_2 \sim \left(\frac{a^4}{\Delta^{13}}\right)^{1/6} \frac{\Theta_D^2}{\lambda T^{3/2}} \exp\left(\frac{\epsilon_0}{T}\right) ,$$

here $\lambda$ is the phonon interaction constant and $\Theta_D$ is the Debye temperature.

Let $\tau_1\omega \ll 1 \ll \tau_2\omega$. Then the electrons and phonons are in equilibrium with one nother during the time of variation of the field, but the number of electronic xcitations is not in equilibrium. Their distribution function takes in this case Fermi form with a chemical potential not equal to zero. Then the terms cor-esponding to the former processes vanish, and the equation can be integrated o yield an expression for the total number of excitations.

Let initially the film be in good contact with the medium and $T$=const. Let $Z$ be the ratio of the number of excitations in the film to their equilibrium value $\zeta_0$ at $A_2 = 0$

$$\frac{dZ}{dt} = \frac{1}{\tau_2} \{\exp(\kappa \cos\omega t) - Z^2\} , \tag{3}$$

vhere

$$\tau_2^{-1} = \frac{2\pi\lambda}{\rho_0 \Theta_D^2}\left[1 + \left(\frac{\epsilon_0}{\Delta}\right)^{2/3}\right]\epsilon_0^2 X^\circ ;$$

$$\kappa = 2\left(\frac{\epsilon_0^{(0)}}{a_0}\right)^{1/3} \frac{a_\omega}{T} .$$

Neglecting the time dependence everywhere except in the exponential, and re-cognizing that $Z$ varies little over the period, we obtain the stationary solution

$$Z = \sqrt{I_0(\kappa) + \frac{1}{\tau_2\omega} \int_0^{\omega t} [\exp(\kappa\cos x) - I_0(\kappa)]\, dx} \tag{4}$$

where $I_0(\kappa)$ is a Bessel function, with $I_0(\kappa) \sim \exp(\kappa)/\sqrt{2\pi\kappa}$ at $\kappa \gg 1$. The number of excitations increased greatly, and the phonons are in equilibrium in this case if the time of disintegration of a phonon into two is $\tau_3 \ll \omega^{-1}$ for energies $\sim 2\epsilon_0$.[4,5]

We consider now a thermally insulated film. An increase in the number of excitations cools the film. The new temperature can then be found from the entropy conservation law. It first decreases sharply in comparison with the ini-tial temperature $T_0$, and then varies slowly during the field oscillation period. The entropy $S(T,t)$ of the excitations is determined as the entropy of the non-equilibrium Fermi gas.[6]

$$S(T,t) = \frac{4\epsilon_o^{(o)}}{T} Z(T,t) X_o(T), \quad (5$$

$$\frac{T_o - T}{T_o} = \frac{S(T_o, t) - S(T_o, 0)}{C(T_o, t)} \quad (6$$

at $T_0 - T \ll T_0$, where $C(T,t)$ is the total heat capacity of the sample. At values $T_0 \sim 1°$, $\Delta \sim 10°$, $\Theta_D \sim 100°$, and other characteristic metal parameters we have $\tau_1$, $\tau_3 \sim 10^{-7}$ sec, $\tau_2 \sim 10^{-5}$ sec, $\omega \sim 10^6$ sec$^{-1}$, and $(T_0 - T)/T_0 \sim 10^{-1}$.

The effect can be easily observed by passing alternating current through a thermally-insulated film.

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