Short-wave dissipative instability on trapped electrons

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It is shown theoretically that a dissipative instability due to the trapped electrons can develop in a plasma contained in a tokamak at wavelengths that are small in comparison with the Larmor radius of the ions. The development of this instability calls for the presence of an electron-temperature gradient larger than the density gradient.

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It is known that in a plasma contained in toroidal magnetic traps of the tokamak type there can develop dissipative instabilities due to the trapped ions[1] and the trapped electrons.[2] Both instabilities are connected with long-wave perturbations, i.e., with transverse wavelengths large in comparison with the Larmor radius of the ions. Greatest interest attaches to the instability due to trapped ions, since it is not suppressed by the shear of the magnetic field. As to the dissipative instability due to trapped electrons,[2] according to [3] it is suppressed already at a relatively small shear. We consider in this connection the question of dissipative buildup of short-wave perturbations by the trapped electrons, the perturbation wavelength being small in comparison with the Larmor radius of the ions.

Just as in[1,2], we assume the perturbations to be potential and use for their description the quasi-neutrality condition $n_i = n_e$, where n_i and n_e are the perturbations of the density of the ions and of the electrons, respectively. Taking into account the already advanced assumption that the perturbations are

short-wave, we conclude that the perturbed ion density is determined by the Boltzmann formula, in other words, $e_i = -e_i \psi n_0 / T_i$, where n_0, T_i, and e_i are the equilibrium plasma density, the ion temperature, and the ion charge, while ψ is the potential of the perturbed electric field E, defined by the relation $\mathbf{E} = -\nabla\psi$. The perturbed density of the electrons will be represented in the form of an integral with respect to the velocity $\mathbf{v}$ and their perturbed distribution function, $f_e = \int f d\mathbf{v}$. The function f consists of two parts $f = f^{(1)} + f^{(2)}$, where $f^{(1)}$ is the "Boltzmann" part, $f^{(1)} = -e_e \psi F / T_e$ (F is the equilibrium distribution func tion of the electrons), and $f^{(2)}$ is the deviation from the "Boltzmann" part. For untrapped electrons, $f^{(2)} = 0$, and for trapped ones, in accordance with[4,5], the function $f^{(2)}$ can be represented in the form $f^{(2)} Fh \exp(iqn\theta)$, where the function h satisfies the equation

$$\omega h - i\bar{C}(h) = (\omega - \hat{\omega}_{*e}) l_e \bar{\psi} / T_e .\tag{1}$$

Here q is the tokamak margin coefficient, θ is the angular coordinate along the minor azimuth of the torus, n is the wave number along the major azimuth of the torus, ω is the oscillation frequency, D is the collision operator, $\hat{\omega}_{*e}$ is the operator of the gradient (drift) frequency of the electron, the explicit form of which is given in[4,5]. The superior bar denotes averaging over the period of the motion of the trapped electrons.

We assume the collisions to be infrequent, $\omega h \gg \bar{C}(h)$. Eq. (1) for the elec tronic h can then be solved in the same manner as the equation for the ionic h was solved in[4]. Taking this analogy into account and using the results of[4], we arrive at expressions of interest to us for the frequency $\text{Re}\omega \equiv \omega_0$ and the increment $\gamma \equiv \text{Im}\omega$ of the short-wave perturbations:

$$\omega_0 = 0.58 \frac{2\epsilon^{1/2} \omega_{*i}}{1 + T_i / T_e} ,\tag{2}$$

$$\frac{\gamma}{\omega_0} = -\frac{2.6(\bar{\nu}_e / \epsilon \omega_0)^{1/2} (l_1 + \eta_e l_2)}{\{ \ln[32(\epsilon\omega_0 / 2\bar{\nu}_e)^{1/2}] \}^{3/2}} ,\tag{3}$$

where

$$(l_1, l_2) = \frac{2}{\sqrt{\pi}} \int_0^\infty dz z^{-1/4} e^{-z} [1 + H(z)]^{1/2} \left(1, z - \frac{3}{2}\right).\tag{4}$$

$\epsilon \equiv a/R$ is the ratio of the minor (running) radius of the tokamak a to the major radius R, $\omega_{*i} = (cT_i / e_i B_s)(\partial \ln n_0 / \partial a)m / a$, m is the wave number in the minor azimuth, B_s is the average equilibrium magnetic field, $\bar{\nu}_e$ is the average fre quency of the electron collisions, [4] $\eta_e \equiv \partial \ln T_e / \partial \ln n_0$, and the explicit form of the function $H(z)$ is given in[4].

The difference between our formulas (2) and (3) and formulas (103) and (90) of[4] is merely that the electron and ion subscripts are interchanged, and that the forms of I_1 and I_2 are different: formula (90) of[4] contains $H^{1/2}(z)$ instead of $(I + H(z))^{1/2}$. This last difference is due to the fact that in the electron dissipa tion case of interest to us an important role is played by collisions of the trap ped particles with the untrapped particles of the same species (with the un-

apped electrons) and with the oppositely-charged particles (ions), whereas in e case of ion dissipation, described by formula (90) of[4], only the collisions the trapped particles with the untrapped ones of the same species (with the itrapped ions) are significant.

Calculation of the integrals I_1 and I_2 yields

$$I_1 = 1.61 , \qquad I_2 = -1.07.\tag{5}$$

'aking this into account, we obtain from (3) the instability criterion

$$\eta_e > 1.52.\tag{6}$$

'his result is analogous to the corresponding result[4] concerning the ion build-p at $\eta_i > 1.75$. The numerical difference between these two criteria is connect-d with the aforementioned difference in the nature of the electron and ion issipation.

We note also that expression (2) for the perturbation frequency is in qualita-.ve agreement with the model analysis of[6] (compare also with[7]).

We have thus shown that in the presence of an electron-temperature gradient condition (6)) there can develop in the tokamak plasma a short-wave dissipa-ive instability due to trapped electrons. An instability of this type, as well as ie long-wave dissipative instability due to trapped electrons, which is widely iscussed at the present time, can lead to an increase of electronic thermal onductivity in tokamaks of future generation (i.e., under conditions of suffi-iently infrequent electron collisions).

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