Temperature dependence of the density of the Bose condensate in liquid ^{4}He

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Inelastic neutron scattering is used to measure the temperature dependence of the density of the Bose condensate in liquid helium. The character of the dependence coincides with the temperature dependence of the density of the superfluid component. The Bose-condensation temperature is $T_0 = T_\lambda$. The relative density o the Bose condensate is $\xi_0\ (p=0,\ T=0) = 0.024 \pm 0.004$.

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The results of investigations of the amount of Bose condensate in liquid helium-4 at temperatures 1.2 and 4.2 °K were discussed in[1] in conjuntion with an analysis of the neutron inelastic-scattering spectra at momentum transfers $\bar{k} = 14.1$ Å^{-1}.

The results presented here were obtained at liquid-helium temperatures $T = 1.2$, 1.8, 2.11, 2.17, 2.35, and 4.2 °K. The experiments were performed with a DIN-IM spectrometer[2] using a pulsed IBR-30 reactor operating with a linear accelerator. The spectra of the scattered neutrons were measured for three scattering angles, for which $\bar{k} = 14.1$, 13.4, and 12.6 Å^{-1}, respectively.

To describe the experimental data we used a two-Gaussian mathematical model

$$\phi(n) = E^2 A_1 \exp(P_1) + E^2 A_2 \exp(P_2) + (A_7 + nA_8), \qquad (1$$

where

$$P_{1,2} = -(E - E_z + A_{3,6})^2 A_{2,5}^{-1} k^{-2}.$$

The second term in (1) describes scattering by the Bose condensate.

The experimental spectra were obtained at a constant scattering angle θ, so that the free-particle energy E_z and k are connected with the initial energy (E_0) and with the final energy (E) of the neutron by the relations

$$E_z = \frac{\hbar^2 k^2}{2M_{\rm He}},$$

$$k = \frac{\sqrt{2M_n}}{\hbar}(E_0 + E - 2\cos\theta\sqrt{E_0 E})^{1/2}.$$

The connection between E and the number n of the time-analyzer channel is of the form

$$E = c(a + bn)^{-2},$$

where a, b, and c are constants determined from the experimental conditions.

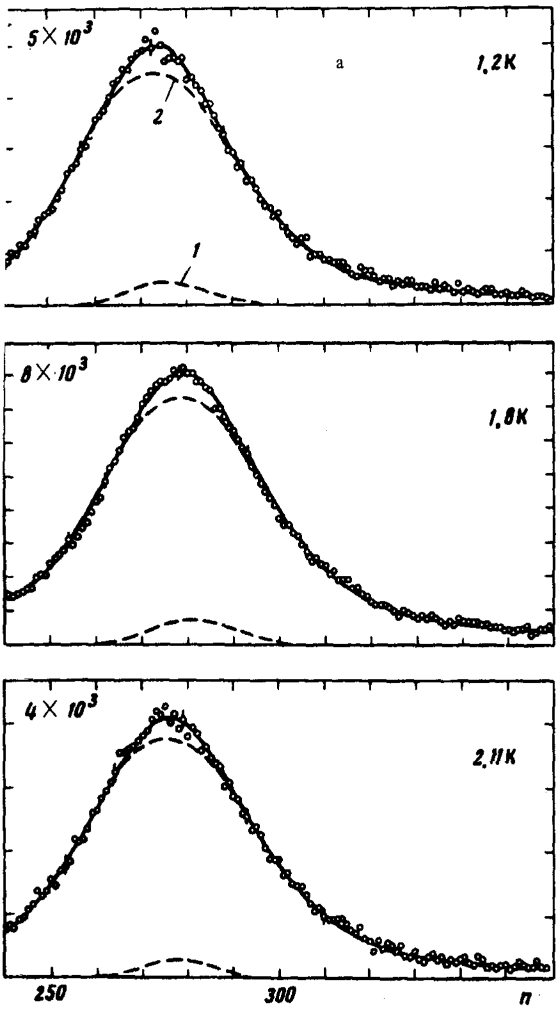

FIG. 1a, b. Experimental spectra of neutrons scattered by liquid helium at $\bar{k}=14.1$ Å^{-1} for different temperatures. The calculated curves pertain to the Bose-condensate (curve 1) and the above-condensate (curve 2) parts, which are shown dashed.

The experimental data were reduced on the basis of the regularized Gauss-Newton iteration process proposed in[3]. The independent parameters were chosen to be the amplitudes A_1 and A_4 of the Gaussian curves, the squares A_2 and A_5 of the widths, the shifts $A_3=A_6$ of the sought Gaussians relative to the free-particle energy, and the straight-line parameters A_4 and A_8. The statistical errors of the parameters were obtained by using least-squares error theory.

The possible causes of the appearance of the third term in (1) are the presence of a maximum with $p\sim 2.5$ Å^{-1} in the spectrum of the helium-atom momentum distribution, [4] or else the presence of a monotonically decreasing part of the spectrum in the region of large momenta.

The relative amount of Bose condensate was calculated from the formula

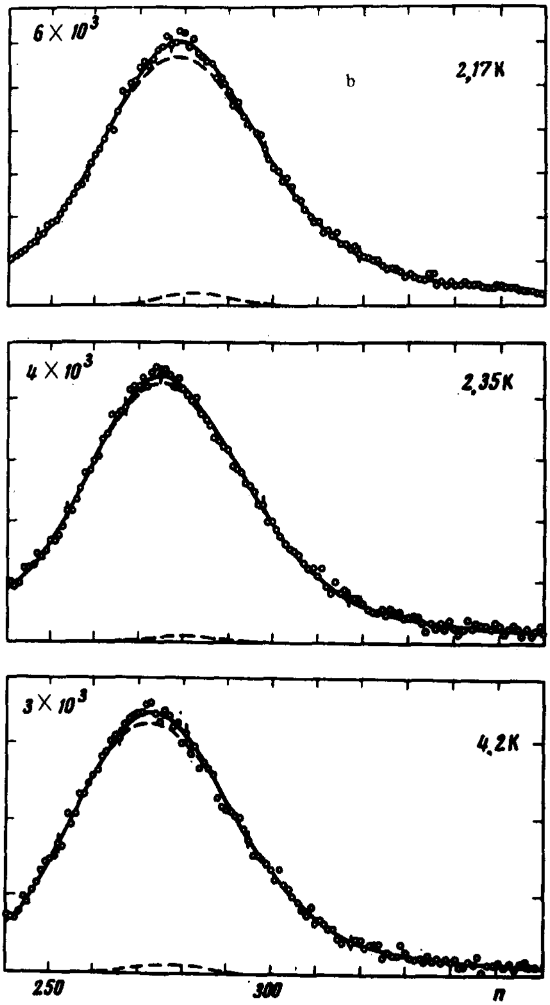

FIG. 1b.

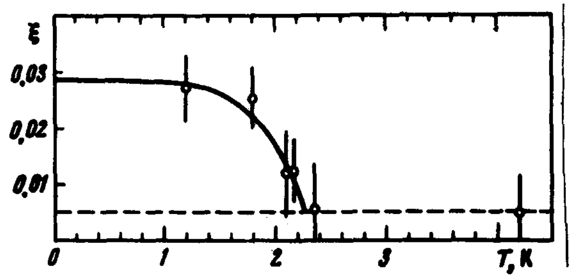

FIG. 2. Temperature dependence of the Bose condensate in liquid helium. The dashed line shows the systematic error, while the solid line is a plot of (2).

$$\xi = \frac{S_{Bc}}{S_{Bc} + S_{ac}} ,$$

where S_{Bc} and S_{ac} are the areas of the spectra for the Bose-condensate and above-the-condensate parts, respectively.

Figures 1a and 1b show the experimental spectra of the neutrons scattered by helium for six different temperatures at $\bar{k} = 14.1$ Å^{-1}.

Figure 2 shows the measured temperature dependence of the amount of Bose condensate in liquid helium. The measured dependence has a singularity at a temperature T_0. At $T < T_0$ an increase in the amount of Bose condensate is observed with decreasing T, and at $T \geq T_0$, up to the boiling point $T = 4.2$ °K of liquid helium, the obtained amount of Bose condensate remains unchanged within the limits of errors. It can be assumed that the Bose-condensate fraction is determined at $T > T_0$ by the systematic measurement error connected with the employed experimental technique and with the method of analyzing the spectra of the scattered neutrons.

The experimental values of A_5, i.e., of the square of the width of the Gaussian for the Bose-condensate part, does not depend on the temperature or on the scattering angle and is equal, within the limits of errors, to the square of the width of the spectrometer resolution function.

The experimental data were analyzed also without allowance for the Bose-condensate part in (1) ($A_4 = 0$). From the point of view of the statistical criterion χ^2/s of the approximation quality, the two-Gaussian model is better at $T < T_0$ and the one-Gaussian model at $T > T_0$.

The experimental results of the temperature dependence of the relative amount of Bose condensate were described with the aid of the empirical formula

$$\xi(p = 0, T) = \xi_0(p = 0, T = 0)\left[1 - \left(\frac{T}{T_0}\right)^m\right] , \tag{2}$$

where ξ_0 is the relative density of the Bose condensate at $T = 0$. The values of the free parameters ξ_0, T_0 and m were determined by least squares:

$$\xi_0 = 0.024 \pm 0.004; \quad T_0 = 2.29 \pm 0.12\text{K}; \quad m = 5.6 \pm 4.$$

The obtained values of the Bose-condensation temperature coincide with the λ-transition temperature of liquid helium: $T = T_\lambda$, [5] and the character of the temperature dependence of the Bose condensate coincides with the temperature dependence of the amount of the superfluid component. [7] The first results on the temperature dependence of the Bose-condensate density, obtained in the present paper, create realistic premises for the investigation of the connection between Bose condensation and superfluidity.

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