Fermion-density phase transitions in models with continuous breaking of the gauge and chiral symmetry groups

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It is shown that the behavior of gauge models with spontaneous symmetry breaking depends, in the limit of high density of matter, on the relation between the constants of the interaction of the vector and spinor particles with the scalar fields. The existence of a density phase transition in the Weinberg model for compression of electrically neutral matter is possible only if the baryon and lepton charges of the substance are equal.

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The question of fermion-density phase transitions in gauge models with spontaneous symmetry breaking has been under discussion many times recently. The first statement of the possibility of such a transition is due to Harrington and Yildiz.[1] As was shown later by Linde[2] in a Higgs model[3] with zero-mass fermions interacting with a massive vector field, when the fermion density is increased an effect opposite to that indicated in[1] takes place, namely, the average scalar field increases montonically. The latter is connected with the fact that in the gauge theories a weak fermion charge induces an opposite weak charge of scalar particles. (This effect, which appears already in the tree approximation, was not taken into account in[1]).

In modern models of weak and electromagnetic interaction, the fermion masses are introduced in invariant fashion via interaction with a scalar field having a nonzero vacuum expectation value. We shall show that in the presence of massive fermions, depending on the relation between the constants of the interaction of the scalar (ϕ), spinor (φ), and vector (A_μ) fields, both situations considered in[1] and[2] can take place. In the Weinberg model, the existence of a phase transition upon compression of electrically neutral matter depends on the ratio of the lepton and baryon charges of the substance.

We illustrate this using as an example spontaneous breaking of $U(1)\otimes U(1)$ gauge symmetry. The Lagrangian of this model is

$$L=-\frac{1}{4}\sum_{j=L,R}\{F_{\mu\nu}(A^j)\}^2+(D_\mu\phi)^+(D_\mu\phi)+\mu^2\phi\phi^+-\lambda(\phi\phi^+)^2$$
$$+\sum_{j=L,R}\bar{\psi}_{(j)}i\gamma_\mu[\partial_\mu-ig_{(j)}A_\mu^{(j)}]\psi_{(j)}-h(\bar{\psi}_L\phi\psi_R+\bar{\psi}_R\phi^+\psi_L), \qquad (1)$$

where $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$, $D_\mu\equiv\partial_\mu-ig_L A_\mu^L+ig_R A_\mu^R$, and the subscripts L and R pertain to fields that transform in accordance with the representations of the left and right symmetry groups.

In the limit of high fermion density, the effective potential of the considered system of fields, in a unitary gauge, is given in the quasiclassical approximation by

$$\Omega = -(g_L^2 + g_R^2)\rho^2 Z_0^2 - \mu^2\rho^2 + \lambda\rho^4 - Z_0(g_L^2 n_L - g_R^2 n_R)/\sqrt{g_L^2 + g_R^2} + \sum_{j=L,R} \int^{k_F^{(j)}} \frac{d^3k}{(2\pi)^3}\sqrt{k^2 + h^2\rho^2}\,, \tag{2}$$

where $\rho = |\phi|$, $Z_\mu = (g_L A_\mu^L - g_R A_\mu^R)/\sqrt{g_L^2 + g_R^2}$ is a combination of vector fields and acquires a mass, $n_{L,R}$ is the density of the fermions with helicity ∓ 1, and $k_F^{(j)} = (6\pi^2 n^j)^{1/3}$ are the limiting Fermi momenta. The first three terms in (2) correspond to the usual tree approximation, and the last term corresponds to the energy of a degenerate gas of left-hand and right-hand fermions with mass $m = h\rho$.

Since the weak charge $Q_W \sim (g_L^2 n_L - g_R^2 n_R)$ of the fermions interacting with the massive vector field is not conserved, it should be obtained in the ground state from the condition that the effective potential be a minimum at a given total number of fermions $n = n_L + n_R$. By varying (2) with respect to all the independent variables Z_0, ρ, and n_L we can easily obtain their dependence on the fermion density n. It is readily seen that in the case of a weak difference between the constants g_L and g_R

$$\left|\frac{g_L}{g_R} - 1\right| < \frac{3h}{4\pi} \tag{3}$$

there exists a critical fermion density n_c, above which the average scalar field is equal to zero

$$n_c = \frac{8\pi}{3}\left(\frac{\;}{h}\right) \frac{\sqrt{2}(g_L^2 + g_R^2)}{\left\{g_L^{4/3} + g_R^{4/3} - 8\pi^2 n^{-1}(g_L^{2/3} - g_R^{2/3})^2\right\}^{3/2}}\,. \tag{4}$$

If the constants are equal, $g_L = g_R$ (when $n_R = n_L = n/2$), the expression for the critical density (4) coincides with that given in[1].

If ratio of g_L and g_R is the inverse of (3), no phase transition takes place. In the limit of a strong difference between the constants, $g_L \gg g_R$, at asymptotically high densities $n \gg (\mu/h)^3$, the average scalar field increases with increasing density

$$\rho(n) = 2^{-7/2}\lambda^{-1/6}\left\{1 - \frac{1}{2}\left(\frac{\lambda}{6\pi^2}\right)^{1/3}\right\} n^{1/3}. \tag{5}$$

In this limit, the numbers of fermions with different helicity differ from each other by an amount

$$n_R - n_L = \frac{3}{2}\left(\frac{\lambda}{6\pi^2}\right)^{1/3} n\,. \tag{6}$$

We note that expressions (4—6) were derived by us under the assumption λ

$> h^2$, which is one of the conditions of the stability of vacuum (see[4] in this connection) in the Higgs model with fermions.

A more realistic model for the description of the effects considered above is the Weinberg model[5] with spontaneous breaking of $SU(2)\otimes U(1)$. Of greatest interest for cosmological problems (the evolution of the universe, gravitational collapse) is the contraction of electrically-neutral matter. This problem can be solved in the Weinberg model with hadrons included.[6] Calculations perfectly analogous to those presented above for the $U(1)\times U(1)$ model, but with a much larger number of variables (densities of leptons and hadrons with different helicity) lead to the following results[1)]: if the lepton and hadron densities differ strongly (for example, in a universe with a large excess of neutrinos),[2 then the average scalar field increases with increasing density. In the case of when the lepton and baryon charges are approximately equal, a density phase transition to a state with restored symmetry of vacuum is possible.

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1)The detailed calculations are quite elaborate and will be published separately.

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