

# Inclusive $p$ collision spectra at high energies

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It is shown that the recently obtained experimental data on the behavior of the inclusive  $pp$  collision spectra in the pionization region agree well with the previously proposed hypothesis of  $z$ -scaling, i.e., with the assumption that the normalized distribution in rapidity depends only on the ratio  $z = y/Y$  in the limit as  $Y \rightarrow \infty$ .

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It is known that the Feynman hypothesis whereby the central region of the inclusive spectra has a uniformly expanding plateau with a height independent of the energy<sup>[1]</sup> is in good agreement with the approximate constancy of the total section and the logarithmic growth of the average multiplicity ( $n \propto Y = \ln(\sqrt{s}/m_p)$ ). Measurements of the total, total inelastic, and elastic cross sections of  $pp$  collisions in a wider energy interval have shown that these cross sections increase by 8–12% and can be successfully parametrized on the basis of an asymptotic form in powers of  $Y$ <sup>[2]</sup>

$$\sigma \propto Y^\beta \quad (1)$$

$Y \rightarrow \infty$

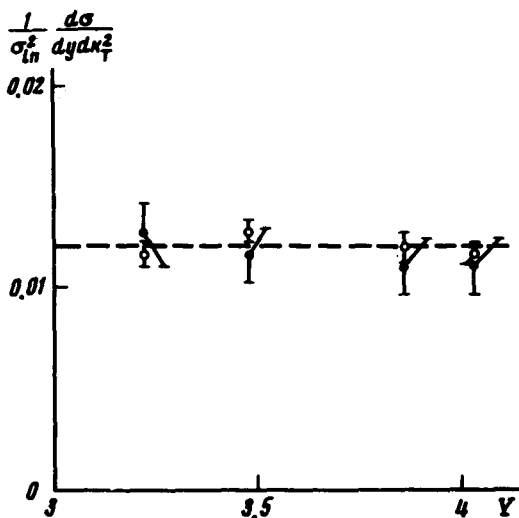


FIG. 1. Normalized height of  $\mu$  meson spectrum in  $pp$  collisions ( $y=0$ ,  $k_T=0.4$  GeV/c):  $\circ$ —data of<sup>[31]</sup>,  $\bullet$ —data of<sup>[101]</sup>, data on  $\sigma_{in}$  from<sup>[111]</sup>. The dashed line corresponds to the optimal value  $(120 \pm 2) \times 10^{-4}$  of the constant in the right-hand side of (4).

It was established practically simultaneously that in  $pp$  collisions the height of the inclusive spectrum

$$\left( \frac{d\sigma}{dy dk_T^2} \Big|_{y=0} \right)$$

increases by 10–15%, so that the reformulation of the Feynman hypothesis in terms of the densities, such as  $(1/\sigma)(d\sigma/dy)$ , did not seem satisfactory. However, the data on the height of the spectrum had a considerable spread, and a

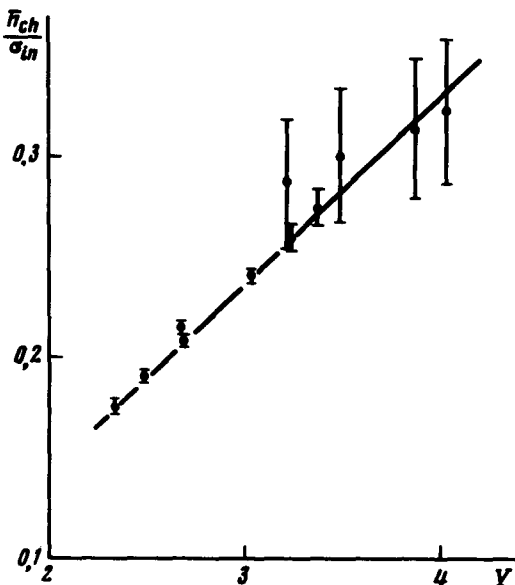


FIG. 2. Ratio of the average multiplicity of the charged particles to the total inelastic cross section,  $n_{ch}/\sigma_{in}=AY+B$ , where  $A=0.095 \pm 0.003$ ,  $B=-(0.048 \pm 0.009)$ , and  $\chi^2/8=0.43$ . The data are from<sup>[12-14]</sup>.

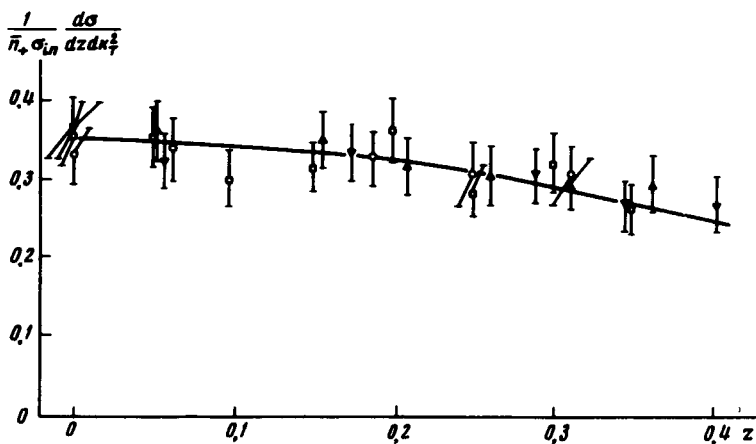


FIG. 3. Check on  $z$ -scaling in accord with the data of<sup>[31]</sup> of the energies ( $\sqrt{s}$ ):  $\circ$ —23.5 GeV,  $\nabla$ —30.6 GeV,  $\Delta$ —44.9 GeV,  $\square$ —52.8 GeV. The line corresponds to parametrization in accordance with formula (7) at  $k_T=0.4$  GeV/c,  $\beta=2$ ,  $C=0.350 \pm 0.004$ , and  $\chi^2/24=0.30$ .

well-argued analysis of the situation called for measurements in the rapidity regions  $0 < y \lesssim 1$  over a wide energy interval.

In this paper we show that the recently obtained data on the inclusive spectra in the region of low rapidities<sup>[31]</sup> make it possible to give preference to a more general hypothesis on the behavior of the soft component of multiple generation- $z$  scaling. This form of scale invariance was proposed in<sup>[41]</sup> in connection with an analysis of the mechanism of the expansion of the Feynman plateau. It turned out that a natural explanation of this expansion can be based on the assumption that a dominant role is played by "fast pionization"—emission of a large number of particles that carry away a finite fraction of the initial rapidity. The limiting form of the  $z$ -scaling hypothesis is

$$\lim_{Y \rightarrow \infty} \left( \frac{1}{Y^\gamma} \frac{d\sigma}{dy dk_T^2} \right) = f(z, k_T) \quad (2)$$

at fixed  $z=y/Y$  and  $k_T$ , where  $\gamma$  is a certain constant, with  $\gamma=0$  for constant total cross sections and  $\gamma=2\beta$  in the general case (1).<sup>[5]</sup>

From (2) it is easy to obtain

$$\bar{n} = \frac{1}{\sigma} \int_{-\gamma}^{\gamma} dy \int dk_T^2 \frac{d\sigma}{dy dk_T^2} \rightarrow \frac{1}{Y^{\beta-\gamma-1}} \int_{-1}^1 dz \int dk_T^2 f(z, k_T^2) \propto Y^{\beta+1} \quad (3)$$

and to derive a number of other directly verifiable corollaries:

$$\lim_{Y \rightarrow \infty} \left( \frac{1}{\sigma_{in}^2} \frac{d\sigma}{dy dk_T^2} \right) \Bigg|_{\substack{y=0 \\ k_T \ll k_T}} = \text{const}, \quad (4)$$

$$\lim_{Y \rightarrow \infty} \left( \frac{\bar{n}}{\sigma_{in} Y} \right) = \text{const}, \quad (5)$$

$$\lim_{Y \rightarrow \infty} \left( \frac{1}{\bar{n}_{oh} \sigma_{in}} \frac{d\sigma}{dz dk_T^2} \right) = \phi(z, k_T^2). \quad (6)$$

The asymptotic form (5) shows clearly that growing cross sections cannot be reconciled with a logarithmic growth of  $\bar{n}$ .

To specify concretely the right-hand side of (6), we used the universal diffraction regime model,<sup>[5]</sup> based on an estimate of the spectra with the aid of Mueller-Kancheli diagrams, with an additional assumption that the usual factoring of the many-point amplitudes is preserved when the leading singularity is chosen in  $j$  plane of all the channels in the form

$$f(j, t_i) = \frac{\text{const}}{t_i = 0 (j-1)^{\beta+1}}$$

This model yields

$$\phi(z, k_T) = C(k_T)(1-z^2)^\beta. \quad (7)$$

The limiting regime (in the sense of the Froissart theorem) with  $\beta=2$  leads to the characteristic asymptotic relations

$$\left. \frac{d\sigma}{dy} \right|_{y=0} \propto Y^4, \quad \bar{n}_{ch} \propto Y^3,$$

which is in good agreement with the experimental data shown in Figs. 1 and 2 with allowance for the known growth of  $\sigma_{in}$  in the same energy interval. Figure 3 shows the  $z$ -scaling form of the inclusive spectrum of  $\pi^+$  mesons at  $k_T=0.4$  GeV/c. It is seen that this scaling is satisfied in a rather wide interval of  $z$  and is in good agreement with the parametrization (7).

It is interesting to note that the Landau hydrodynamic theory predicts for the same rapidity region a qualitatively different scaling with respect to the variable  $u = y/\sqrt{Y}$ .<sup>[6]</sup> For example,  $z$ -scaling results from the model of strongly interacting pomerons,<sup>[7]</sup> but calculation of the exponent, within the framework of the Wilson  $\epsilon$ -expansion method, yields  $\beta=1/6$ . Yet an estimate of the best value of  $\beta$  from the form of the spectrum (7) leads to  $\beta_{opt} = 1.61 \pm 0.23$  at  $\chi^2/23 = 0.28$ , i. e., the limiting regime turns out to be almost optimal. We note also that according to preliminary estimates the limiting regime describes well the behavior of the spectra at other values of  $k_T$ , and makes it possible to use analogous parametrizations for other reactions. A particularly clear fact is the rapid growth of the average multiplicity,  $\bar{n} \propto Y^3$ , in  $p p$ ,  $\pi p$ , and  $k p$  collisions. For example, in the first case  $\bar{n}_{ch} = (AY+B)^3$ , where  $A = 0.32 \pm 0.01$  and  $B = 1.01 \pm 0.02$ . We emphasize that the possibility of a cubic and not linear (in  $Y$ ) universal growth of the average multiplicity was indicated by many authors on the basis of various model representations.<sup>[8,5,9]</sup>

A detailed analysis of the  $z$ -scaling hypothesis and a comparison with all the principal experimental data will be published separately.

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