Penetration of electromagnetic field through a metal plate in the case of diffuse electron reflection

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We consider the influence of the character of electron reflection from the surface on the passage of a doppleron wave through a metallic plate in a constant magnetic field perpendicular to the plate. It is shown that the amplitude of the transmitted signal in strong fields is much larger for diffuse reflection than for specular reflection. A qualitative explanation of the physical nature of this effect is presented.

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It is known[1] that under the conditions of the anomalous skin effect the impedance of a metal depends little on the character of the electron reflection from the surface. Nor does the reflection exert a significant influence on the excitation of a helicon in a plate.[2] The examination of the electromagnetic properties of metals under conditions of diffuse reflection is a much more complicated problem than in the case of specular reflection. The authors of most papers on the theory of penetration of an RF field through a metal have therefore confined themselves to specular reflection, assuming that the diffuseness of reflection does not lead to qualitative changes in the result. This assumption turns out to be incorrect. When a doppleron or a Gantmakher-Kaner "wave" is excited, the transmitted signal may be much stronger for diffuse reflection of the electrons than for specular reflection. We demonstrate this with a simple model that admits of an exact analytic solution.

We consider a compensated metal, the electronic part of the Fermi surface of which constitutes two paraboloid cups with joined rims,[3] and the hole part is a cylinder with an axis parallel to the axis of the electron lens. The constant magnetic **H** and the normal to the plane of the surface (the z axis) are parallel to the symmetry axis of the Fermi surface. In this model, the nonlocal conductivity has no branch point, so that there is no Gantmakher-Kaner effect. In the case of diffuse reflection, the distribution of the electric field in the plate, for "minus" circular polarization corresponding to the direction of electron rotation in a magnetic field, is determined by the equation

$$\frac{d^2E(\zeta)}{d\zeta^2} = -\frac{\xi}{2\pi}\int_0^L d\zeta' E(\zeta')\int_{-\infty}^{+\infty} dq\left[\frac{1-i\gamma}{(1-i\gamma)^2-q^2}-1\right]e^{iq(\zeta-\zeta')} \quad (1)$$

with boundary conditions

$$\left(1+\frac{1}{iq_0}\frac{d}{d\zeta}\right)E(\zeta)\Big|_{\zeta=0} = 2E_0, \quad \left(1-\frac{1}{iq_0}\frac{d}{d\zeta}\right)E(\zeta)\Big|_{\zeta=L} = 0. \quad (2)$$

re $\zeta = 2\pi z/u$, $L = 2\pi d/u$, d is the plate thickness, $u = (c/eH)\,\partial S/\partial p_z$ is the dis-
acement of the electrons over a cyclotron period, S is the area of intersection
the Fermi surface and the plane p_z is the longitudinal component of the elec-
on momentum, $\xi = \omega\omega_p^2 u^2/4\pi^2\omega_c c^2$, ω is the frequency of the exciting field with
plitude E_0, while ω_p and ω_c are the plasma and cyclotron frequencies of the
ectron; $\gamma = \nu/\omega_c$, ν is the frequency of the collision of the electrons with the
tice, and $q_0 = \omega u/2\pi c$. The boundary conditions correspond to incidence of
e external wave on the left-hand side of the plate.

The Fourier component of the kernel of (1) is a rational function of q, and
nsequently this equation reduces to a differential equation with constant co-
icients. The corresponding dispersion equation has two solutions q_s and q_D,
aracterizing the skin and doppleron components of the field. For a thick
ate ($\mathrm{Im}\,q_s L \gg \mathrm{Im}\,q_D L > 1$), the solution of (1) and (2) can be written in the form

$$E(\zeta) = \frac{2q_0 E_0}{1+q_D+q_S}\left\{\frac{1+q_S}{q_S-q_D}e^{iq_S\zeta} - \frac{1+q_D}{q_S-q_D}e^{iq_D\zeta} + \frac{(q_S+q_D)(1-q_D+q_S)(1+q_D)}{(q_S-q_D)(1+q_D+q_S)(1-q_D)}e^{iq_D L}\left[\frac{1+q_D}{q_S-q_D}e^{iq_D(L-\zeta)} - \frac{2q_D(1+q_S)}{(q_S^2-q_D^2)(1-q_D+q_S)}e^{iq_S(L-\zeta)}\right]\right\}. \quad (3)$$

In the case of specular reflection,[4] under the same assumptions, we obtain

$$E_{sp}(\zeta) = \frac{2q_0 E_0}{q_S^2-q_D^2}\left\{\frac{q_S^2-1}{q_S}e^{iq_S\zeta} - \frac{q_D^2-1}{q_D}\left[e^{iq_D\zeta} + e^{iq_D(2L-\zeta)}\right]\right\}. \quad (4)$$

The first and second terms in (3) describe the skin-effect and doppleron
ld components propagating from the the left-hand surface of the plate, while
e third term describes the field of the doppleron reflected from the right-
nd surface. Similar terms are contained in (4). In addition, in (3) there is
ourth term that describes the skin-effect component that is produced on
e right-hand side surface as a result of the doppleron reflection. In (4) there
no such component, i.e., no skin layer is produced when the doppleron is
flected from the right-hand boundary in the specular case. In addition, the
plitudes of the corresponding components in (3) and (4) differ appreciably.
eir difference is manifest particularly in the region of moderately strong
gnetic fields, a region defined by the inequalities

$$\gamma \ll \xi \ll 1, \quad (5)$$

which the approximate expressions for q_S and q_D are

$$q_S \approx (1+i)\sqrt{\gamma\xi/2}, \qquad q_D \approx -1 + \frac{\xi}{2} + i\gamma. \quad (6)$$

It is easy to verify that in the region (5) the impedance of a semiinfinite
ctal is $Z_\infty^{sp} \approx 4\pi q_0/c\,q_s$ in the case of specular reflection and $Z_\infty \approx 8\pi q_0/c\xi$

in diffuse reflection. The doppleron amplitude also depends strongly on the character of the reflection. Its value is $2q_0E_0$ in (3) and $2q_0E_0$ in (4), i.e., it is much larger in diffuse reflection than in specular reflection. However, the role of diffuse reflection is not confined to this. The magnitude of the signal $E(L)$ passing through the plate is determined not by the electronic field of the doppleron but by the field of the skin layer produced on the right-hand surface Indeed, the amplitude of the fourth term in (3) is larger by ξ^{-1} times than the amplitude of the transmitted doppleron. Thus, the magnitude of the field $E(L)$ having the doppleron phase depends fundamentally on the character of the elec tron reflection: $E(L)/E_{sp}(L) \approx \xi^{-2} \gg 1$.

The physical cause of the result can be defined in the following manner. In the case of specular reflection, the electrons bring in the doppleron field and the same electrons carry away the same field. The electric field of the transmitted and reflected dopplerons add up on the right-hand surface, and their magnetic fields cancel each other practically completely, so that no skin laye is produced. In diffuse reflection, on the other hand, the electron loses the information concerning the field. Therefore, the field produced on the right-hand surface by the transmitted doppleron can be regarded as an external source for the reflected electrons that carry the field back into the metal. Under the influence of this source, the reflected electrons form at the right-hand surface a skin layer and a reflected doppleron, similar to what occurs o the left surface of the plate under the influence of the external wave. It is see from (3) that the amplitude of the reflected doppleron is smaller than the amplitude of the transmitted one by an amount on the order of q_S/ξ. The electric field of the skin component, on the other hand, is smaller by a factor q_S/q_0 than the magnetic field, i.e., it is $1/\xi$ times larger than the electric field on the doppleron (the latter is smaller by a factor $1/q_0$ than the magnetic field of the doppleron). Similar reasoning can explain also the cause of the more effective excitation of the doppleron on the left-hand boundary in the cas of diffuse reflection of the electrons.

It is obvious that these effects will manifest themselves also in the impedan of a plate that is excited on both sides. In addition, it is natural to expect the difference between the character of the reflection to manifest itself not only in doppleron propagation, but also in the propagation of Gantmakher-Kaner "waves."

[1]G.E.H. Reuter and E.H. Sondheimer, Proc. R. Soc. A, **195**, 336 (1948).
[2]G.A. Baraff, Phys. Rev. **178**, 1155 (1969).
[3]R.G. Chambers and V.G. Skobov, J. Phys. **F1**, 202 (1971).
[4]D.S. Falk, B. Gerson, and J.F. Carolan, Phys. Rev. **B1**, 406 (1970).