lany-particle Regge poles

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It is shown that in the ϕ^3 theory there exist Regge poles made up of many-particle states in the t channel. Their intercept increases quadratically with the number of particles in this channel.

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It has long been customary to connect with the reggeon an aggregate of ladder iagrams, even though the $g\phi^3$ theory leads to an interecept of the pole $\alpha(0)$ ear $j=-1$.

The problem of obtaining a positive intercept on account of many-particle :ates in the t channel has in this connection always been enticing.

An important stage in the development of the Regge scheme was the analysis : nonplanar ladder diagrams with three and four particles in the t channel,[1] hich has lead to the concept of branch points in the j plane.

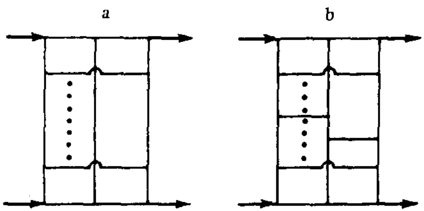

FIG. 1.

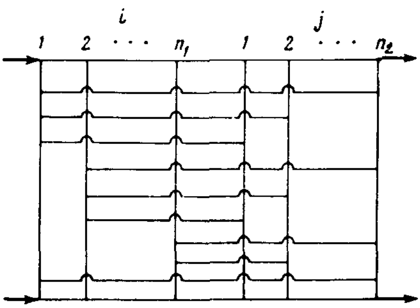

FIG. 2.

It it obvious, however, that the Mandelstam diagrams corresponding, for example, to four particles in the t channel are not sufficient. They correspon(to configurations when the particles, interacting pairwise, form two "bound" states—reggeons. In principle, in a theory with attraction, bound states of three and four particles are possible in a system of four particles, and this circumstance must be taken into account. This problem was considered first in field theory by McCoy and Wu[2] who investigated, in the principal (with respect to the logarithms of s) ϕ^3-theory approximation, the asymptotic form of the ladder diagrams with three particles in the t channel (Fig. 1). They have shown that the contribution to the asymptotic form of the diagrams of typ Fig. 1(b) dominates over the contribution of the Mandelstam diagrams of type 1(a) corresponding to reggeon—particle branch cuts.

This important result requires generalization to the case of an arbitrary number of particles in the t channel, and this is the subject of the present communication.

In connection with the obvious complexity of the problem we resort not to th Feynman parametrization used in[2] for the integrals, but to the technique of the Sudakov parameters,[3] which is relatively simple and easier to visualize.

Assume that the t channel contains n_1+n_2 particles, of which n_1 emit and n_2 absorb an arbitrary number of particles. Figure 2 shows an example of the diagrams considered by us, where certain crosspieces of importance to the asymptotic forms are drawn.

A detailed analysis shows that from the point of view of the principal asymptotic form in $\ln s/m^2$, any line of the group n_1 can be joined to any line of the group n_2, leading thereby to an additional factor $g^2 \ln s/m^2$. On the other hand, connections between lines inside each of the groups are inessential in the principal logarithmic asymptotic form, leading each time to factors on the order of g^2.

Let L be the number of all the essential rungs. Then the order of the diagra is $2(L+n_1+n_2)$, the number of the vertical propagators is $2L+n_1+n_2$, and that of the horizontal ones is $L+2(n_1+n_2-1)$.

In the principal logarithmic approximation we can obtain for the asymptotic form of a diagram with L crosspieces the following expression (q is the trans verse momentum):

$$- i \frac{s^{-n_1 - n_2 + 1}}{(L + m_1)!} g^{2(L + n_1 + n_2)} \ln^{L + m_1}(s/m^2) K(q), \tag{1}$$

here

$$m_{1(2)} = max\ (min)\ \{n_1, n_2\}.$$

Here $K(q)$ is the "transverse" integral corresponding to contraction of all ıe horizontal lines of the diagram into points. Its estimated lower bound ($q = 0$) ;

$$K(0) > (2\pi)^{n_1 + n_2 - 2} \left(\frac{1}{2(2\pi)^3}\right)^{L + n_1 + n_2 - 1} \left(\frac{\pi\gamma^2}{m^2}\right)^{L + 1}$$

$$\times [K_0(2\gamma)]^{2L + n_1 + n_2}, \tag{1'}$$

here $K_0(x)$ is a Macdonald function and γ is an arbitrary constant. Adding now .l the topologically different diagrams of order $2(L + n_1 + n_2)$ obtained by permu- .tion of the crosspieces, and also all the possible crossings of the vertical nes both within the groups n_1 and n_2 and of the lines of group n_1 with lines of roup n_2, and summing over all the crosspieces, we obtain ultimately for the ggregate of diagrams with $n_1 + n_2$ particles in the t channel the asymptotic ›rm

$$- i \left(\frac{g^2}{m^2}\right)^{m_2} [K_0(2\gamma)]^{m_2 - m_1} \left(\frac{2}{\gamma^2}\right)^{m_1 - 1} \left(\frac{1}{8\pi^2}\right)^{m_2 - 1}$$

$$\times \left(\frac{s}{m^2}\right)^{a^{(n_1, n_2)}(0)} \sigma_{n_1 n_2}, \tag{2}$$

here the position of the singularity is determined by the inequality

$$a^{(n_1, n_2)}(0) \geqslant - n_1 - n_2 + 1 + n_1 n_2 \frac{g^2}{16\pi^2} \frac{\gamma^2}{m^2} K_0^2(2\gamma), \tag{3}$$

r

$$\sigma_{n_1 n_2} = \left[1 + \exp\left(i\pi\left(-1 + \frac{g^2}{16\pi^2} \frac{\gamma^2}{m^2} K_0^2(2\gamma)\right)\right)\right]^{n_1 n_2} \tag{4}$$

: the signature factor.

The most essential result is the appearance in (3) of a term with $n_1 n_2$ mean- .g that allowance for many-particle states in the t channel can lead, in princi- .e, to a positive intercept even in a field theory with scalar particles.

Formulas (2)—(4) show that we are dealing with $n_1 n_2$ effective ladders that ·e interconnected along the entire length of their side pieces.

In spite of this coupling, each of the ladders retains its individuality, a fact ›flected in the form of the signature factor (4).

The singularity (3) is a pole, since in impact-parameter space the particles .at couple the ladders along their entire length cannot move away a distance .rger than $1/m$, so that the ladders do not diverge in this space at any rapidity.

Thus, if we speak of the picture of the pomeron in the language of Feynman diagrams, it is not described at all by ladder-type diagrams and is determined more readily by the generalized multiperipheral configurations considered here which contain a large number of coupled ladders (see also[5], where a phenomenological picture of a heavy pomeron is developed). This circumstance can be essential for many characteristics of multiple production. The question of the presence of the $n_1 n_2$ effect in other theories with attraction calls for a special analysis.

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1)After this work was completed, two preprints appeared[4] in which the result of[2] was obtained with the aid of the Sudakov parameters for three and four particles in the t-channel.

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