Reggeization of the fermion in non-Abelian gauge theories

V. S. Fadin and V. E. Sherman

Nuclear Physics Institute, Siberian Division, USSR Academy of Sciences
(Submitted April 8, 1976)
Pis'ma Zh. Eksp. Teor. Fiz. **23**, No. 10, 599–602 (20 May 1976)

We calculate the high-energy behavior of the backward scattering amplitude of a vector meson by a fermion in spontaneously broken Yang-Mills model and find that the fermions become reggeized. Branch points are present, in addition to the pole, in channels with negative signature.

PACS numbers: 11.80.Cr

Field-theoretical models based on Yang-Mills gauge fields have attracted much attention of late.[1] The vanishing of the interaction at short distances at non-Abelian gauge theories leads to an approximate Bjorken scaling,[2] thus stimulating the use of such theories for the description of strong interactions. The interest in the high-energy behavior of the amplitudes in these theories is therefore natural. A study of the forward scattering amplitudes, initiated in[3], proved the reggeization of the vector meson[3–5] and explained the position and character of the vacuum singularity.[5]

We consider in this paper the amplitude of the scattering of a vector meson by a fermion at scattering angles close to 180°. The high-energy behavior of this amplitude permits an explanation of the question of the reggeization of the fermion. For simplicity we confine ourselves here to a very simple model[6] based on an isotriplet of Yang-Mills vector fields $\mathbf{V}^{\mu}$ with mass m, produced a a result of the appearance of a nonzero vacuum mean value for the isodoublet complex scalar field. The interaction between the fields $\mathbf{V}^{\mu}$ and an isodoublet of fermions of mass M takes the form $-g\bar{\psi}\hat{\mathbf{V}}(\vec{\tau}/2)$.

The amplitude $A_{AB}^{A'B'}$ of the process $A+B\to A'+B'$, where A and A' are vecto mesons and B and B' are fermions, is calculated in the principal logarithmic approximation

$$g^2 \ln\frac{S}{m^2} \sim 1, \qquad g^2 << 1; \quad S = (p_A + p_B)^2 >> m^2, \quad -t = -q^2 \sim m^2. \qquad (1)$$

Here $q = p_{B'} - p_A$; the components of q that do not vanish as $S\to\infty$ lie in a plane perpendicular to p_A and p_B; $q \approx q_\perp$. In the approximation (1), the amplitude $A_{AB}^{A'B'}$ can be represented in the form

$$A_{AB}^{A'B'} = g^2 \bar{u}(p_{B'})\left(\hat{e}_A - \frac{2\hat{p}_A(e_A p_B)}{S}\right)\left\{\frac{\tau^a}{2}\frac{\tau^{a'}}{2}(A_{1/2}^{+} + A_{1/2}^{-}) + \frac{1}{2}\left(\delta^{aa'} - \frac{\tau^a\tau^{a'}}{3}\right)A_{3/2}^{-}\right\}\left(\hat{e}_{A'} - \frac{2\hat{p}_{A'}(e_{A'}p_{B'})}{S}\right)U(p_B), \qquad (2)$$

by separating contributions with definite isospin in the t channel and with positive and negative signature. We note that the positive-signature part, which exceeds the negative signature, in each order of perturbation theory in $\ln S$ because of the cancellation in the latter of the contributions of the S and U channels, is present only in the channel with the quantum numbers of the fermion. This circumstance is the consequence of the commutation relations and is valid not only for the considered model, but also for groups of higher order. For the amplitudes $A_T^{\pm}$ we use the j representation ($\omega = j - 1/2$)

$$A_T^{\pm}(S; q) = \frac{1}{4i}\int_{\delta - i\infty}^{\delta + i\infty} d\omega \left(\frac{S}{m^2}\right)^{\omega} \frac{(e^{-i\pi\omega} \pm 1)}{\sin\omega\pi} F_T^{\pm}(\omega, q). \qquad (3)$$

In the calculation of the amplitude $A_{AB}^{A'B'}$, just as in the forward scattering problem, it is convenient to use the dispersion method[3-5]. Our calculations, up to eight order perturbation theory, have shown that in this approximation $F_{1/2}^{+}$ is represented by a Regge pole with a trajectory

$$j = \frac{1}{2} + \delta(q_\perp); \qquad \delta(q_\perp) = \frac{3g^2}{4(2\pi)^3}(\hat{q}_\perp - M)\int \frac{d^2 k_\perp}{(M - \hat{k}_\perp)(m^2 - (q-k)_\perp^2)}, \qquad (4)$$

passing through $j = 1/2$ at $\hat{q}_\perp = M$, which means reggeization of the fermion, whereas for the negative signature the partial amplitudes cannot be represented as an expansion of Regge poles. We investigated the higher orders of perturbation theory by two methods. One of them made it possible to calculate directly, in each order, the senior logarithmic term, which contributes only to $A_{1/2}^{+}$, and to confirm the reggeization of the fermion. The other method is analogous to that used in[5] and its gist is the following: the dispersion approach requires nowledge of the amplitudes in order to reconstruct $A_{AB}^{A'B'}$ by unitarity and analyticity. In the kinematics that makes the main contribution to the unitarity elations, the inelastic amplitudes calculated by us for order g^8 have a simple multiregge form and can be easily generalized to an arbitrary order. Using hem, we reconstruct $A_{AB}^{A'B'}$. If we represent $F_T^{\pm}(\omega, q)$ in the form (from now on ll the vectors are two-dimensional and perpendicular to p_A and p_B)

$$F_T^{\pm}(\omega, q) = \frac{r_T^{\pm}}{M - \hat{q}} + C_T^{\pm} \frac{g^2}{(2\pi)^3} \int \frac{d^2k}{(m^2 - (q-k)^2)(M - \hat{k})} f_T^{\pm}(\omega; k, q) \quad (5)$$

here

$$r_{1/2}^{+} = 1; \quad r_{1/2}^{-} = r_{3/2}^{-} = 0; \quad C_{1/2}^{+} = -\frac{3}{4}; \quad C_{1/2}^{-} = \frac{25}{12}; \quad C_{3/2}^{-} = 2. \quad (6)$$

hen we obtain for $f_T^{\pm}(\omega; k, q)$ the equation

$$[\omega - a((q-k)^2) - \delta(k)] f_T^{\pm}(\omega; k, q) = 1 + \frac{g^2}{(2\pi)^3} \int \frac{d^2k'}{(m^2 - (q-k')^2)}$$

$$\times \left\{ a_T^{\pm} \left[\hat{q} - M + (M - \hat{k}) \frac{1}{M - (\hat{k} + \hat{k}' - \hat{q})} (M - \hat{k}') \right] + b_T^{\pm} \left[\hat{q} - M + (M - \hat{k}) \right.\right.$$

$$\left.\left. \times \frac{(m^2 - (k'-q)^2)}{(m^2 - (k'-k)^2)} + (M - \hat{k}') \frac{(m^2 - (k-q)^2)}{(m^2 - (k'-k)^2)} \right] \right\} \frac{1}{M - \hat{k}'} f_T^{\pm}(\omega; k', q). \quad (7)$$

Iere

$$a_{1/2}^{\pm} = \mp \frac{1}{4}; \quad b_{1/2}^{\pm} = 1; \quad a_{3/2}^{-} = b_{3/2}^{-} = -\frac{1}{2} \quad (8)$$

$a(q^2) + 1$ is the trajectory of the vector meson[3]

$$a(q^2) = \frac{g^2}{(2\pi)^3}(q^2 - m^2) \int \frac{d^2k}{(m^2 - k^2)(m^2 - (q-k)^2)}. \quad (9)$$

It is easy to verify that in the case of $T = 1/2$ and of positive signature the solution of (7) is

$$f_{1/2}^{+}(\omega; k, q) = \frac{1}{\omega - \delta(q)}. \quad (10)$$

Then

$$F_{1/2}^{+}(\omega, q) = \frac{1}{M - \hat{q}} \frac{\omega}{\omega - \delta(q)}; \quad A_{1/2}^{+} \approx \frac{(S/m^2)^{\delta(q)}}{M - \hat{q}}. \quad (11)$$

It is impossible to solve Eq. (7) for negative signature. It can only be stated that the corresponding partial amplitudes have besides the poles generated by the two-particle unitarity condition in the t channel also branch points due to the exchange of reggeized particles—the vector meson and fermion. Other types of singularity are also possible; thus, for example, the mechanism that seems to lead to the appearance of an immobile branch point in the vacuum channel[5] (existence of arbitrarily high thresholds with respect to t) operates here, too.

We note in conclusion the following: the problem of reggeization of the fermion was first solved in quantum electodynamics with a massive photon in[7]. Our equations can lead to the quantum-electrodynamics results by leaving out from (2) the term with $A^{-}_{3/2}$ and replacing

$$\frac{\tau^a}{2}\frac{\tau^{a'}}{2}$$

by unity; in (5) and (7) it is necessary to make the substitutions

$$C^{\pm}_{T} \to \mp 1; \quad a^{\pm}_{T} \to \pm 1; \quad \delta(k) \to \delta_1(k) = \frac{4}{3}\delta(k),$$

and to set $\alpha((q-k)^2)$ and $b^{\pm}_{T}$ equal to zero in accordance with the fact that the photon is not reggeized and there is no three-photon interaction. Then formulas (10) and (11) remain valid with the substitution $\delta(q) \to \delta_1(q)$, and show that the fermion is reggeized. However, in the channel with negative signature and in quantum electrodynamics, the partial amplitude is not represented by a Regge pole. Our results for this amplitude coincide here with those obtained recently in[8] and show that the statement made in[7] concerning the degeneracy with respect to signature is incorrect.

We are grateful to E. A. Kuraev, L. N. Lipatov, and L. L. Frankfurt for interest in the work and for valuable discussions.

[1]C. H. Yang and R. Mills, Phys. Rev. **96**, 191 (1954).
[2]D. C. Gross and F. Wilczek, Phys. Rev. Lett. **26**, 1343 (1973); H. D. Politzer, Phys. Rev. Lett. **26**, 1346 (1973).
[3]L. N. Lipatov, Preprint LIYaF No. 157, 1975; Yad. Fiz. **23**, 642 (1976) [Sov. J. Nucl. Phys. **23**, No. 3 (1976)].
[4]L. L. Frankfurt and V. E. Sherman, Preprint LIYaF No. 186, 1975; Yad. Fiz. **23**, 1101 (1976) [Sov. J. Nucl. Phys. 23, in press (1976)].
[5]V. S. Fadin, E. A. Kuraev, and L. N. Lipatov, Phys. Lett. **60B**, 50 (1975); E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, Zh. Eksp. Teor. Fiz. **71**, No. 3 (1976) [Sov. Phys.-JETP **44**, No. 3].
[6]G. t Hooft, Nucl. Phys. **B33**, 173 (1971); **B35**, 167 (1971).
[7]M. Gell-Mann, M. L. Goldberger, F. E. Low, E. Marx, and F. Zachariaren, Phys. Rev. **133B**, 145 (1964).
[8]B. M. McCoy and T. T. Wu, Phys. Rev. Lett. **35**, 1190 (1975).