New type of cyclotron waves in metals

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It is shown, by numerically solving the dispersion equation for extraordinary cyclotron waves propagating in silver, that a new type of such waves exists. These new waves depend on the presence of orbits of carriers of both signs. The dispersion properties, the damping, and the polarization of the new waves are obtained.

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Kaner and Skobov[1] have shown that high-frequency electromagnetic waves —cyclotron waves—can propagate in metals in the presence of a magnetic field oriented in the manner required for the observation of cyclotron resonance. They have considered a spherical Fermi surface (FS). Several calculations of the dispersion equation of cyclotron waves were made later on[2–4] for different FS models at $\omega\tau=\infty$, where ω is the radiation frequency and τ is the relaxation time.

A realistic case of the FS of silver and a finite value of $\omega\tau$ were considered in[5,6]. We have recently noted[8] that the dispersion equation for the cyclotron wave can be regarded as the spatial condition for the minimum of the damping due to collisions. It was subsequently shown[9] that the exact solution of the dispersion equation for the cyclotron waves in the case of a cylindrical FS is determined by zeroes of Bessel functions. We note that the calculation procedure of[9] can be extended to other FS models.

We shall show in this communication that in the case of an anisotropic FS, similar to the FS of silver,[7] the dispersion equation for the cyclotron waves of the extraordinary type admits of the existence of waves of a new type in addition to those considered earlier.[5,6,8,9] It turns out that the existence of these waves calls for the presence of orbits of both signs—electron and hole—and in this respect the new waves recall Alfven waves; they are, however, generally speaking elliptically polarized in a plane perpendicular to the magnetic field **B**. In analogy with previously known waves, the dispersion equation of the wave of the new type has several branches; we consider below only the case of extremely long waves.

The dispersion equation for the wave vector $\mathbf{q} \parallel x$, $\mathbf{B} \parallel z$, and $\mathbf{E}_{rf} \parallel y$ is of the form

$$q^2\sigma_{xx} = (4\pi i\omega/c^2)(\sigma_{xx}\sigma_{yy} + \sigma_{xy}^2), \tag{1}$$

where the components of the magnetic conductivity tensor σ_{ij} can be calculated by the Chambers formula[10] for the FS of silver.[7] The numerical calculations were performed with a computer in accordance with the program developed in[5,6]. We consider the case when the vector $\mathbf{B} \parallel \langle 100\rangle$ and lies in the (011) plane, and $\mathbf{q} \parallel \langle 011\rangle$. For this magnetic-field orientation, the FS of silver can

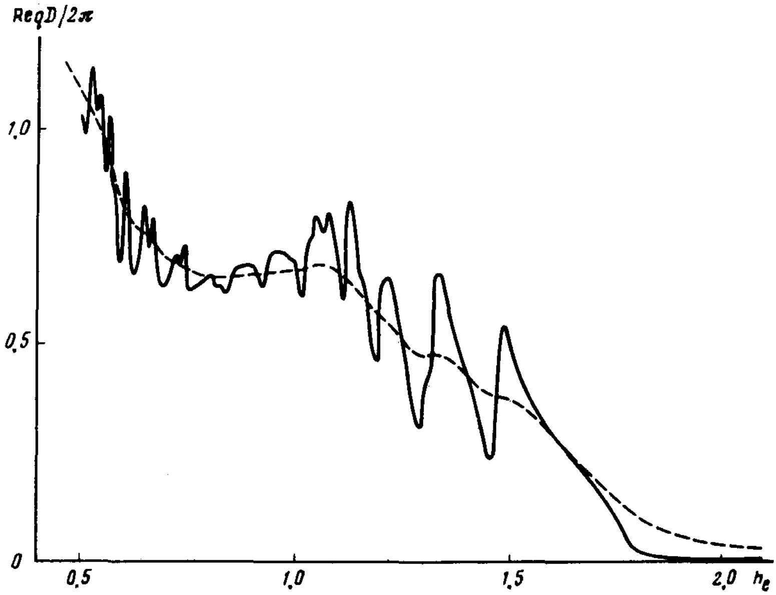

FIG. 1. Plot of $\mathrm{Re}(qD/2\pi)$ against $h_e = \omega_{ce}/\omega$ for the new type of waves in the long-wave limit. Dashed—45 GHz, solid—300 GHz. The calculations are based on the Halse model of the Fermi surface of silver. $\mathbf{B} \parallel \langle 100 \rangle$; $\mathbf{q} \parallel \langle 011 \rangle$.

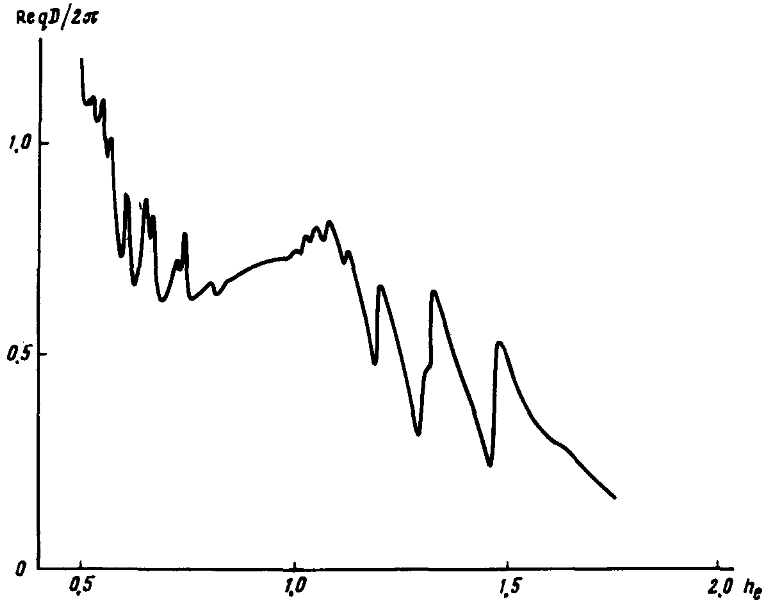

FIG. 2. Dependence of $\mathrm{Re}(qD/2\pi)$ on h_e for the new type of waves. The calculations are based on the Halse model for the FS of silver with allowance for the spherical electron cap; $\mathbf{B} \parallel \langle 100 \rangle$; $\mathbf{q} \parallel \langle 011 \rangle$; frequency 300 GHz.

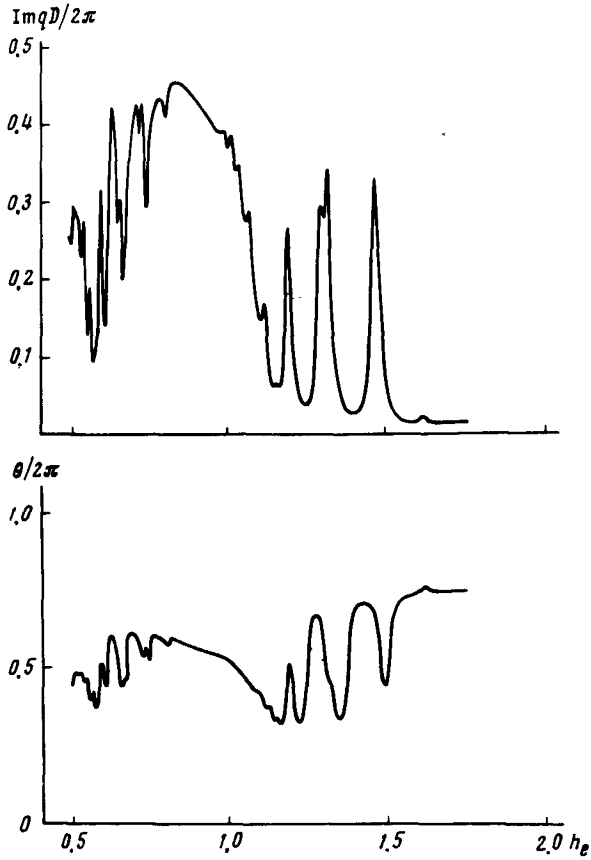

FIG. 3. The lower part shows the phase angle between the longitudinal and transverse components of the field as a function of h_e. The upper part shows the function $\mathrm{Im}(qD/2\pi)$ vs h_e. The calculation conditions are the same as for Fig. 2.

be regarded as consisting of three parts: 1) a central cylinder-like part that gives electron cyclotron orbits of diameter D and cyclotron frequency ω_{ce}; 2) a part that gives the hole orbits (fourth rosette) of cyclotron frequency ω_{ch}; 3) spherical cap of electron orbits. The calculations were performed for two frequencies, 45 and 300 GHz, assuming $\omega\tau = 20$ at 45 GHz and $\omega\tau = 135$ at 300 GHz. These values correspond to $\tau = 7\times10^{-11}$ sec and are typical of our orbits with the silver samples. The results of the calculations are presented in the form of plots of $qD/2\pi$ against $h_e = \omega_{ce}/\omega$ and of the phase angle θ against h_e, with $E_x/E_y = \alpha\exp(i\theta)$.

Figure 1 shows the function $\mathrm{Re}(qD/2\pi)$ against h_e for the waves of the new type at 45 GHz (dashed) and 300 GHz (solid), obtained with the entire FS of silver taken into account.[7] We see that both the values and the behavior of the function $\mathrm{Re}(qD/2\pi)$, which are obtained from the new solution, distinguish this solution noticeably from those previously obtained (see, e. g., [8]). To assess the gist of the new solution, we have tried to exclude from the calculation the hole part of the FS, and it turned out that in this case no numerical solution can be obtained for $qD/2\pi$ as a function of h_e. Both types of carrier are thus needed if the cyclotron wave of the new type is to appear.

Figure 2 shows also the function $\mathrm{Re}(qD/2\pi)$ against h_e, but this time the calculations were made with the FS of silver without the spherical electron cap taken into account. Comparison with Fig. 1 shows that electrons belonging to the spherical cap of the FS do not influence the dispersion relation significantly, with the exception of the field region $0.85 \lesssim h_e \lesssim 1.1$. This part of the FS of silver will henceforth be disregarded in the subsequent calculations.

The lower part of Fig. 3 shows the phase angle θ between the longitudinal (E_x) and the transverse (E_y) components of the field E, while the upper part shows the quantity $\mathrm{Im}(qD/2\pi)$ as a function of h_e at 300 GHz. It can be noted again that the behavior of the variables differs noticeably from that obtained earlier for the known types of cyclotron waves (see, e.g., [5,8]).

The oscillating character of the quantities $\mathrm{Re}(qD/2\pi)$, $\mathrm{Im}(qD/2\pi)$ and θ as functions of the magnetic field is obvious from Figs. 1—3. The amplitude of the oscillations decreases with decreasing frequency, but remains noticeable at 45 GHz, for example, in the case of $\mathrm{Im}(qD/2\pi)$ (not given here). The mean value of $\mathrm{Im}(qD/2\pi)$ reaches a maximum somewhat below $h_e=1$, and then decreases sharply above $h_e=1$.

It can be noted that the oscillations shown in Figs. 1—3 have a tendency to break up into two groups; this is seen particularly clearly in Figs. 2 and 3. The oscillations of one group appear at $h_e=1.62$ and vanish somewhat above $h_e=1$, whereas the oscillations of the second group appear at $h_e=0.81$ and attenuate near $h_e=0.5$. Each maximum of the oscillations of the first group has a twin in the second group at half the value of the magnetic field. We propose that these oscillations are determined by an equation of the type

$$(\omega_{ce}+\omega_{ch})n=\omega m \quad (n \text{ and } m \text{ integers}). \qquad (2)$$

Using the ratio $\omega_{ce}/\omega_{ch}=1.15$,[11] we find that $(n,m)=(1,2)$ and $(1,3)$ correspond to the values $h_e=1.07$ and 1.61, respectively. These values of h_e seem to be the limits of the first group of oscillations, as is seen in the lower part of Fig. 3. The limits of the second group (and of the subsequent groups) follow automatically from Eq. (2). Furthermore, the values $m/n=2.75$, 2.5, and 2.25 lead to values h_e lying quite close to the three strong maxima of the function $\mathrm{Re}(qD/2\pi)$, which are shown in Fig. 2. A detailed discussion of this circumstance and of other questions will be given later.

It should be noted in conclusion that an experimental calculation of the existence of the new type of cyclotron waves may be hindered by the strong and rapid oscillations of the function $\mathrm{Im}(qD/2\pi)$.

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