Stimulated Mandel'shtam-Brillouin scattering (SMBS) in an inhomogeneous plasma

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Absolute SMBS instability has been observed in a plasma and, in contrast to convective instability, contributes to anomalous heating of the plasma.

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Parametric conversion of electromagnetic radiation (pump wave) into transverse electromagnetic and ion-sound waves (SMBS) attracts much attention of investigators, since it can be the cause of an anomalously large scattering of the incident radiation by a plasma. This conclusion can be drawn, in particular, from the results of the theory of SMBS in a spatially-homogeneous plasma (see, e.g., [1]). We report in this communication the results of a theory of SMBS for a spatially inhomogeneous plasma, which predicts parametric conversion accompanied by the onset of radiation that is trapped in the plasma and cannot leave it. This SMBS conversion leads not to scattering but to absorption of the pump wave, and corresponds to absolute instability in an inhomogeneous plasma (cf. [2,3]).

To analyze the consequences of Maxwell's equations and of the associated equations of the hydrodynamics of non-isothermal electron-ion plasma, we used the geometrical-optics approximation. Then, assuming that the pump-wave field varies little over the characteristic length of variation of the plasma perturbations, we obtained the following expression for the projection $k_x(x)$ of the wave vector on the x-axis, along which the plasma is inhomogeneous:

$$k_x^2(x) \equiv k_\pm^2 = k_\perp^2\{u + v \pm [(u - v)^2 + \frac{1}{4} v_E^2(x) v_{Te}^{-2} \omega_{Le}^2(x) k_\perp^{-2} c^{-2}]^{1/2}\},$$

where $k_\perp$ is the projection of the wave vector on a plane perpendicular to the x axis, v_{Te} is the thermal velocity of the electron, c is the velocity of light in vacuum, $\omega_{Le}(x)$ is the Langmuir frequency of the electrons. Finally, $2u = [\omega^2 - k_\perp^2 v_s^2 + 2i\gamma_s\omega](k_\perp v_s)^2$, $2v = [(\omega-\omega_0)^2 + 2i\gamma_t(\omega-\omega_0) - k_\perp^2 c^2 - \omega_{Le}^2(x)](k_\perp c)^{-2}$, where ω is the frequency of the ion-sound wave, ω_0 is the frequency of the pump wave, v_s is the speed of sound in the plasma, and γ_s and γ_t are the damping decrements of the ion-sound and transverse plasma waves.

The dependence of the electron-oscillation velocity $v_E(x) = eE_0(x)m_e^{-1}\omega_0^{-1}$ on the coordinate, for a linear density profile, when $\omega_{Le}^2(x) = \omega_0^2[1 + xL_N^{-1}]$, is determined by the fact that for the pump-wave field in vacuum we have $E_0(x) = AE(0)\mathrm{Ai}(\zeta)$. Here $E(0)$ is the amplitude of the pump wave in vacuum, $A = 2(\omega_0 L_N c^{-1})^{1/6}$ is a factor that takes into account its swelling in an inhomogeneous plasma, $\mathrm{Ai}(\zeta)$ is an Airy function and $\zeta = xL_E^{-1}$, where $L_E = (c^2 L_N/\omega_0^2)^{1/3}$. This coordinate dependence makes possible the appearance of singular points of the vector $k_x(x)$ with different real parts, a decisive factor for the existence of absolute SMBS instability. The two types of singular points correspond to two different mechanisms of spatial localization of the perturbations, namely, reflection of the perturbations from turning points at which $k_-(x) = 0$, and mutual transformation of the waves at branch points at which $k_+(x) = k_-(x)$, respectively.

We turn first to the case of localization of the perturbations by turning points. If the dimension of the inhomogeneity of the plasma is such that

$$L_N > c\,\omega_0^{-1}(64|\zeta_m|^3)^{-1}\omega_s^{3/2}\gamma_s^{-3/2}\,\omega_0\,\omega_{Le}^{-1}(x_m)\,\omega_0^{3/2}(\gamma_t(c))^{-3/2}$$

$$\equiv |\zeta_m|^{-3}L_1,$$

where $\gamma_t(c)$ is the damping decrement of the transverse wave at the critical point, then the SMBS threshold is described by a formula close to that obtained for a homogeneous plasma[1] ($\omega_s \equiv k_\perp v_s$):

$$\frac{v_E^2(x_m)}{v_{Te}^2} = 16\frac{\gamma_s\gamma_t(c)}{\omega_s\,\omega_0}\left[1 - |\zeta_m|\frac{L_E}{L_N} + \frac{(2n+1)\pi}{4(\pi+4)}|\zeta_m|^{1/2}\left(\frac{c}{\omega_0 L_N}\right)^{1/3}\left(\frac{\omega_0}{\gamma_t(c)}\right)^{1/2}\frac{\omega_0}{\omega_{Le}(x_m)}\right]. \qquad (1)$$

Since the third term is larger than the second in the region where this formula is valid, the SMBS threshold does not decrease on moving along the density profile in the region of the tenuous plasma (as would be expected on the basis of the theory of the homogeneous plasma[1]), but increases.

The SMBS threshold increases as the density profile becomes steeper. At

$$|\zeta_m|^{-3}L_1 > L_N > c\,\omega_0^{-1}\zeta_m^6\gamma_s^{3/2}\omega_s^{-3/2}\,\omega_0^{3/2}(\gamma_t(c))^{-3/2}\omega_0^3\,\omega_{Le}^{-3}(x_m) \equiv \zeta_m^6 L_2$$

the instability threshold is determined by the formula

$$v_E^2(x_m)v_{Te}^{-2} = \zeta_m^{-2}\omega_{Le}^2(x_m)\omega_0^{-2}(c\,\omega_0^{-1}L_N^{-1})^{2/3}. \qquad (2)$$

For an even steeper profile, when $L_N < \zeta_m^6 L_2$, the threshold increases to a value

$$\frac{v_E^2(x_m)}{v_{Te}^2} = 2\zeta_m^2 \frac{\gamma_t \gamma_s}{(\gamma_t + \gamma_s)^2} \frac{\omega_{Le}^2(x_m)}{\omega_0 \omega_s}\left(\frac{c}{\omega_0 L_N}\right)^{4/3}\left[1 + \sqrt{1 - \left(\frac{\gamma_s + \gamma_t}{\gamma_t}\right)^2 |\zeta_m|^{-3}}\right]. \qquad (3)$$

For example, if the plasma produced by bombarding a D_2 target with an Nd laser has a temperature $T_e \approx 1$ keV and an inhomogeneity dimension $L_N = 10^{-2}$ cm, then the minimal threshold, described by (2) is equal to 10^{13} W/cm^2 and is reached at $|\zeta_m| = 3.248$.

We proceed now to the case of localization of the perturbations by branch points. If $L_N > c\omega_0^{-1}\omega_0^{3/2}(\gamma_t(c))^{-3/2}$, then the threshold is determined by the dissipation of the plasma waves:

$$\frac{v_E^2(x_m)}{v_{Te}^2} = 4|\zeta_m|\frac{L_E}{L_N}\left[\frac{\gamma_s}{\omega_s} + \frac{\gamma_t(c)}{\omega_0}\,\frac{\omega_{Le}^4(x_m)}{\omega_0^4}\,\frac{L_N}{|\zeta_m| L_E}\right]^2. \qquad (4)$$

It follows therefore that the minimal threshold, which is equal to

$$v_E^2(x_m) v_{Te}^{-2} = 16\gamma_s \gamma_t(c)\omega_s^{-1}\omega_0^{-1}, \qquad (5)$$

is reached at an extremum of the pump field with coordinate $|\zeta_m| \approx \gamma_t(c) \times \omega_s \gamma_s^{-1} L_N L_E^{-1}$. To the contrary, at $L_N < L_3$ the instability threshold depends essentially on the plasma inhomogeneity

$$\frac{v_E^2(x_m)}{v_{Te}^2} = \frac{\omega_{Le}^2(x_m)}{\omega_0^2}|\zeta_m|^{-2}\frac{L_E}{L_N} + \frac{(2n+1)^2 2\pi^2}{(4+\pi)^2}|\zeta_m| \qquad (6)$$

$$\times \left|\frac{\gamma_s}{\omega_s} - \frac{\gamma_t(c)}{\omega_0}\,\frac{\omega_{Le}^4(x_m)}{\omega_0^4}\,\frac{L_N}{|\zeta_m| L_E}\right|^{1/2} \frac{c^2}{\omega_{Le}^2(x_m) L_E^2}.$$

Comparing formulas (1) and (4) we can see that when the plasma inhomogeneity dimension is near the critical point $L_N > L_1$ the minimum threshold (5) is realized in the region of the profile $|\zeta_m| < \gamma_t(c)\omega_0^{-1} L_N L_E^{-1}$, and also at $|\zeta_m| \approx \gamma_t(c)\omega_0^{-1}\omega_s\gamma_s^{-1} L_N L_E^{-1}$. The localization of the perturbations is connected in this case with the turning points or the branch points, respectively. Analogously, comparing formulas (2), (3), and (6) we arrive at the conclusion that at $L_3 > L_N > L_2$ the minimal threshold is described by formula (6) [near the critical point, the threshold (2) of the instability localized by the turning points coincides with that described by formula (6)]. Finally, if $L_N < L_2$, then the instability connected with the turning point, with a threshold (3), is possible only near the critical point. On the remaining profile, the minimal SMBS threshold is again described by formula (6).

Inasmuch as the growing perturbations are localized near the extrema of the pump field, this instability does not lead to anomalous reflection, but contributes to anomalous heating of the plasma. At the same time, a tunnel effect can take place, whereby the satellite wave can emerge from the plasma. If

Δx is the dimension of the instability localization region, then the amplitudes E_e and E_i of the satellite wave outside and inside the localization region are connected by the relation $E_e \approx E_i \exp[-L_E(\Delta x)^{-1}]$. Since the localization region Δx is significantly smaller than the dimension L_E of the pump-wave inhomogeneity, it follows that $E_i \gg E_e$. This allows us to conclude that the reflection due to the tunnel effect is exponentially small.

Absorption of the pump wave (for example as a result of bremsstrahlung, linear transformation, or other parametric instabilities) does not change the absolute character of the investigated instability, but increases the threshold amplitude $E_{thr}(0)$ of the pump wave in vacuum by a factor $R^{-1/2}$, where R is the effective pump-wave reflection coefficient ($R < 1$).

[1] L.M. Gorbunov, Zh. Eksp. Teor. Fiz. **55**, 2298 (1968) [Sov. Phys.-JETP **28**, 1220 (1969)].
[2] F.W. Perkins and J. Flick, Phys. Fluids **14**, 2012 (1971).
[3] V.P. Silin and A.N. Starodub, Zh. Eksp. Teor. Fiz. **66**, 176 (1974); **67**, 2110 (1974) [Sov. Phys.-JETP **39**, 82 (1974); **40**, 1047 (1975)]; Preprint FIAN SSSR, No. 182, 1975.