Conductivity and Hall effect of pure type-II superconductors at low temperatures

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The conductivity and Hall angle of pure type-II superconductors at $T \ll \Delta$ is calculated. These quantities depend strongly on the parameter $\tau\delta\epsilon$, where $\delta\epsilon$ is the distance between the low-lying bound levels in the vortex core. Viscous flow of the vortices corresponds to $\tau\delta\epsilon \ll 1$, and nondissipative flow to $\tau\delta\epsilon \gg 1$.

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Larkin and Ovchinnikov[1] calculated the conductivity in the mixed state at $T \ll \Delta$, when the electron free-path time is in the range $\Delta^{-1} \ll \tau \ll E_F/\Delta^2$, namely $\sigma_f = 0.23\sigma_n H_{c2}/B$. There was no Hall effect under these assumptions.

In the limit as $\tau \to \infty$, at low temperature, the vortex flow should obviously be nondissipative. Since $\dot{\mathbf{E}} = c^{-1}\mathbf{B}\times\mathbf{u}$, the vortex velocity $\mathbf{u}$ should be parallel to the transport current. In this paper we calculate the conductivity and the Hall angle at $T \ll \Delta$ and $\tau\Delta \gg 1$. The transition from the viscous flow of vortices to nondissipative flow is defined by the parameter $x \sim \tau\delta\epsilon$, where $\delta\epsilon \sim \Delta^2/E_F$ is the distance between the low-lying levels in the core of the vortex.[2] An attempt to take this effect into account semiphenomenologically was made in[3]. We shall neglect the influence of the magnetic field, i.e., we assume $H \ll H_{c2}$ and $\kappa \gg 1$. We can subsequently lift the restriction on κ.

We represent the regular Green's functions in the form

$$G^{R(A)}_{(\mathbf{r}_1, \mathbf{r}_2)} = \int \frac{dk}{2\pi} \sum_\nu \exp\left[i\left(\nu + \frac{1}{2}\right)(\phi_1 - \phi_2) + ik(z_1 - z_2)\right] G^{R(A)}_\nu(\rho_1, \rho_2)$$

with analogous expressions for $F^{+R(A)}$ etc. The functions G and F^+ are defined by two linear inhomogeneous second-order equations. They can be expressed in terms of linear combinations of four fundamental solutions of the corresponding homogeneous equations, two of which are finite at the origin, and the two others decrease at $\rho \gg \xi$ (we recall that $\epsilon \sim T \ll \Delta$). At $\tau\Delta \gg 1$, the result takes the form

$$\hat{G}^{R(A)}_\nu(\rho_1, \rho_2) = -\frac{m}{4\psi^{R(A)}} \begin{pmatrix} u(\rho_1)\, u(\rho_2) & -u(\rho_1)\, v(\rho_2) \\ v(\rho_1)\, u(\rho_2) & -v(\rho_1)\, v(\rho_2) \end{pmatrix}, \quad (1)$$

where $\hat{G}$ is the matrix Green's function,

$$\psi^{R(A)} = \frac{m}{q} \int_0^\infty \left(\epsilon + \frac{i}{2\tau} g_0^{R(A)} + \frac{\nu|\Delta|}{q\rho} \right) e^{-2K}\, d\rho,$$

$$q^2 = p_F^2 - k^2, \qquad K = \int_0^\rho \frac{m|\Delta|}{q}\, d\rho, \qquad \hat{g}_0 = (\pi i \nu(0))^{-1} \hat{G}(\mathbf{r}, \mathbf{r}).$$

$$u = e^{-K} J_{|\nu + 1/2|}(q\rho), \quad v = e^{-K} J_{|\nu - 1/2|}(q\rho) \cdot \mathrm{sign}\,\nu .$$

Equation (1) contains only the pole part, corresponding to bound states. It can be verified with the aid of (1) that the principal role in the expression for the transport current is played only by the anomalous functions. Therefore, according to[4,5],

$$\frac{\pi}{e} [\mathbf{j}_{tr} \times \mathbf{n}_H] = \int d^2\mathbf{r} \frac{d\epsilon}{4\pi i} \mathrm{Sp}[(\nabla \hat{H}) \hat{G}^{(a)}] \quad (2)$$

where n_H is a unit vector along the magnetic field

$$\hat{H} = \begin{pmatrix} 0 & -\Delta \\ \Delta^* & 0 \end{pmatrix},$$

and the anomalous function is defined by the equation

$$\hat{G}^{(a)}(\mathbf{r}, \mathbf{r}') = \int d^3\mathbf{r}_1 \hat{G}^R(\mathbf{r}, \mathbf{r}_1) \left[\frac{i}{2T} \mathrm{ch}^{-2} \frac{\epsilon}{2T} (\mathbf{u}\nabla) \hat{H}(\mathbf{r}_1) + \frac{i}{2\tau} \hat{g}_0^{(a)}(\mathbf{r}_1) \right] \hat{G}^A(\mathbf{r}_1, \mathbf{r}'). \quad (3)$$

The main contribution in the summation over ν in (3) is made by the residue of the pole in (1), and the contribution of the nonpole terms left out of (1) is small. As a result we have $g_0^{(a)} = \hat{g}_0^{(a)} = 0$, and the functions

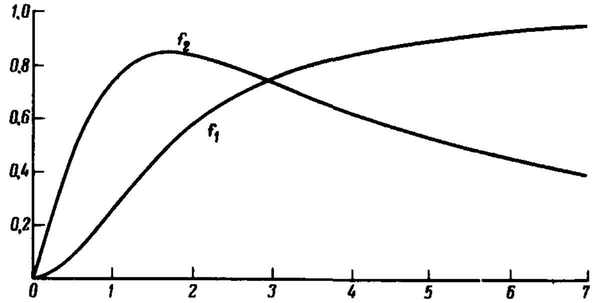

FIG. 1.

$$e^{-i\phi} f_0^{(a)} = -e^{i\phi} f_0^{+(a)} = i \sum_{a=\pm 1} < C_a(q) e^{-2K} > e^{ia\phi} ,$$

where $C_a(q)$ satisfy the integral equation

$$\int_0^\infty \left[C_a(q) \left(\frac{a|\Delta|}{q\rho} + \frac{i}{\tau} < \gamma > \right) e^{-2K} + \frac{i}{\tau} \gamma < C_a(q) e^{-2K} > \right] d\rho \tag{4}$$

$$= -\frac{au}{2T} \, \mathrm{ch}^{-2} \frac{\epsilon}{2T} \, \frac{\pi q^2}{2m} ,$$

and

$$\gamma = \frac{\pi}{2} \left(\frac{\partial \psi}{\partial \nu} \right)^{-1} e^{-2K} , \qquad < > = p_F^{-1} \int \frac{dk}{2\pi q \rho} .$$

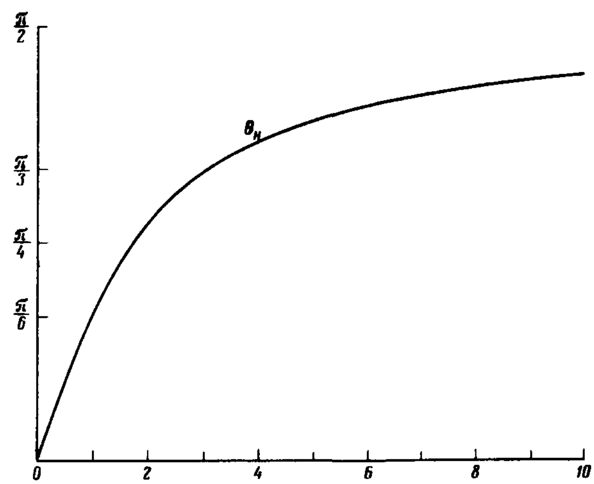

FIG. 2.

Equation (4) has an exact solution at $x \gg 1$. In this case $\mathbf{j}_{tr} = Ne\mathbf{u}$, i.e., the vortex is dragged completely by the superfluid stream. At arbitrary values of the parameter x, Eq. (4) can be solved by using the result of[6], according to which $|\Delta(\rho)|$ assumes its equilibrium value over distances on the order of $\xi_1 = \xi T/\Delta$. We can then lift the restriction on κ, with accuracy logarithmic in Δ/T, and assume that $\kappa \sim 1$. From (2) and (4) we obtain

$$\mathbf{j}_{tr} = Nef_1(x)\mathbf{u} + \frac{3\pi Ne}{16} f_2(x)[\mathbf{n}_H, \mathbf{u}].$$

The conductivity is $\sigma_f = (3\pi Nec/16B)f_2(x)$, and the Hall angle is

$$\theta_H = \operatorname{arc\,tg}[16 f_1(x)/3\pi f_2(x)].$$

Here $x = 4\Delta^2\tau\ln(\Delta/T)/\pi E_F$, and f_1 and f_2 are expressed in terms of elementary functions:

$$f_1(x) = \frac{3}{4}\,\frac{z}{(1+y)^2 + 4z^2/\pi^2x^2}, \qquad f_2(x) = 2x\left[1 - \frac{1+y}{(1+y)^2 + 4z^2/\pi^2x^2}\right],$$

$$y(x) = \frac{2}{\pi}\int_0^\pi \frac{\sin^4\theta\, d\theta}{\sin^2\theta + x^2}, \qquad z(x) = x^2\int_0^\pi \frac{\sin^3\theta\, d\theta}{\sin^2\theta + x^2}.$$

Figure 1 shows plots of $f_1(x)$ and $f_2(x)$, while Fig. 2 shows a plot of $\theta_H(x)$. As $x \to \infty$, we get $f_1 = 1$ and $f_2 = 2.94/x$. As $x \to 0$ we have $f_1 = 3x^2/8$ and $f_2 = x$. The conductivity is then given by the expression obtained in[1].

Experiments on the Hall effect in alloys ($\tau\Delta \lesssim 1$) yield $\theta_H \sim 10^{-3}$. In pure Nb, however (see, e.g., [7]), $\tan\theta_H \sim 1$ and is practically independent of the magnetic field. A more detailed comparison is made difficult by the fact that the data of[7] pertain to $T = 4.2$–$7.8\,^\circ$K. To our knowledge, the behavior of $\theta_H(x)$ at $T \ll \Delta$ has not been investigated.

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