Right-hand currents and the $\Delta T=1/2$ rule in nonleptonic decays of strange particles

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The influence of strong interaction at short distances on nonleptonic decays is calculated in models with right-hand bare currents. The contributions of the corresponding terms of the Hamiltonian $H(\Delta S=1)$ to the matrix elements of the $K\to 2\pi$ decays is estimated.

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In connection with the discovery of new particles in e^+e^- collisions and the observation of anomalous events in neutrino interactions, weak-interaction models based on introduction of new quarks and new currents into the theory are presently intensively discussed.

In[1], in particular, they discuss the $V+A$ current in the interaction of heavy c quarks with the ordinary (d,s) quarks:

$$j_\mu \sim \bar{C}_L\gamma_\mu(-d_L\sin\theta_C)+\bar{C}_R\gamma_\mu S_R\sin\phi+\ldots, \tag{1}$$

where the subscripts R and L refer to right- and left-polarized fermions, θ_C is the Cabibbo angle, and $\sin\phi$ is a new parameter, assumed in[1] to be of the order of unity.

The known $\Delta T=\frac{1}{2}$ rule in nonleptonic decays of strange particles is connected in[1] with the existence of the right-hand component $\bar{C}_R\gamma_\mu S_R$.

In this article we present the results of the calculation of the effects of short-range strong interactions for the Hamiltonian of nonleptonic decays with allowance for the right-hand current, and estimate the matrix elements of the corresponding operators. According to our results, the new currents do not play an essential role in the nonleptonic decays of the strange particles.

Introduction of right-hand currents leads to the appearance of the following operators in the nonleptonic Hamiltonian

$$H_{eff}(\Delta S=1)]=G\sqrt{2}\sin\theta_C\sin\phi[C_T T+C_{B1}B_1+C_{B2}B_2], \tag{2}$$

where

$$B_1=\frac{4}{3}\bar{S}_R d_L\bar{C}_L C_R;\qquad B_2=2\bar{S}_R t^a d_L\bar{C}_L t^a C_R;$$

$$T=i m_c\bar{S}_R\sigma_{\mu\nu}t^a d_L b^a_{\mu\nu}.$$

Here $b^a_{\mu\nu}$ is the gluon-field intensity tensor, t^a are Gell-Mann SU(3) matrices acting in color space and normalized by the condition $\mathrm{Tr}\{t^a t^b\}=2\delta^{ab}$, and m_c is the mass of the heavy quark.

In the calculation of the matrix elements of the Hamiltonian (2) the operators $B_{1,2}$ make no contribution, inasmuch as the admixture of c quarks in ordinary hadrons is small. Heavy quarks can be produced only in short time intervals and at short distances. Inasmuch as in quantum chromodynamics (the theory according to which the strong interaction is connected with an octet of colored massless gluons) the quark-gluon constant is small at short distances, it follows that the contribution of the heavy quarks can be consistently taken into account by summing the diagrams by the renormalization-group method, and reduces to a determination of the coefficient c_T in relation (2).

In the lowest order of perturbation theory, the coefficient c_T is equal to

$$c_T^{(1)} = g/16\pi^2 \, , \tag{3}$$

if the vertex of the interaction of the gluon with the quarks takes the form $\frac{1}{2} g\bar{\psi} t^a \gamma_\mu \psi b_\mu^a$ (b_μ^a is the gluon field). In the higher orders of perturbation theory there appear logarithmic terms $\sim g^3 \ln(m_c^2/m^2)$, $g^3 \ln(m_w^2/m_c^2)$, where m_w is the mass of the intermediate boson and m is the characteristic mass of the strong interactions, $m \sim (m_\pi$ to $m_\rho)$. These terms can be summed over all orders of perturbation theory.

For the virtual momenta $m < p < m_c$, the operator T is diagonal. The influence of the strong interaction is characterized here by the anomalous dimensionality γ_T, which can be obtained by explicitly calculating all the one-lepton diagrams with the operator T as one of the vertices. Such a calculation yields

$$\gamma_T = -2\frac{g^2}{16\pi^2}\left(-\frac{2}{3} + \frac{b}{2}\right), \quad b = 11 - \frac{2}{3}N \tag{4}$$

(N is the number of species of quarks).

We emphasize that in this region one must not include in the anomalous dimentionality of the operator T the anomalous dimensionality of the mass, inasmuch as at momenta smaller than m_c there are no logarithmic corrections to the mass operator of the c quarks. The anomalous dimensionality of the mass comes into play in the momentum region $p > m_c$. Turning on this dimensionality corresponds to making in (4) the substitution

$$-\frac{2}{3} + \frac{b}{2} \to -\frac{2}{3} - 4 + \frac{b}{2} \quad . \tag{5}$$

In addition to the trivial modification (5), it must be recognized that in this momentum region there takes place a mixing of the operators B_1, B_2, and T. The mixing is determined by two-loop diagrams. The matrix of the anomalous dimensionalities takes the form $\gamma(g) = \gamma_0(g) + \gamma_1(g)$, where

$$\gamma_0(g) = \frac{-2g^2}{16\pi^2}\begin{bmatrix} 8 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -\frac{14}{3} + \frac{b}{2} \end{bmatrix}, \quad \gamma_1(g) = \frac{-2g^3}{(16\pi^2)^2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{2}{3} & \frac{5}{3} & 0 \end{bmatrix} . \tag{6}$$

The solution of the equation of the renormalization group for the column of coefficients $\{C_{B1}, C_{B2}, C_T\}_{x=m_w^{-1}}$ can be written in the form[2]

$$\begin{Bmatrix} C_{B1} \\ C_{B2} \\ C_T \end{Bmatrix}_{x=m_w^{-1}} = \mathcal{G} \exp \left\{ - \int_{g(m)}^{g(m_w)} \gamma(g) \frac{dg}{\beta(g)} \right\} \begin{Bmatrix} 1 \\ 1 \\ g(m_w)/16\pi^2 \end{Bmatrix}, \tag{7}$$

where $\beta(g) = -bg^3/16\pi^2 + \cdots$ and the symbol $\mathcal{G}$ denotes the operation of $\mathcal{G}$-ordering, analogous to the usual T-ordering operation: the larger g, the more to the left should the matrix $\gamma(g)$ be located. The integral in (7) actually breaks up into two integrals: in the region $g(m) > g > g(m_c)$ the matrix $\gamma(g)$ is determined from formula (4), while in the region $g(m_c) > g > g(m_w)$ it is determined by (6). In the first region, the integration is trivial and leads to the factor

$$[g^2(m)/g^2(m_c)]^{-2/3b \pm 1/2}.$$

The contribution of the second region to the coefficient C_T can be easily found by expanding the exponential in (7) in powers of γ_1 and "disentangling" the $\mathcal{G}$-product. The final result is

$$C_T = \frac{g(m)}{16\pi^2} \eta_T;$$

$$\eta_T = \left[\frac{g^2(m)}{g^2(m_c)}\right]^{-2/27} \left[\frac{g^2(m_c)}{g^2(m_w)}\right]^{-\frac{14}{3b}} \left\{ 1 + \frac{1}{19}\left[\left[\frac{g^2(m_c)}{g^2(m_w)}\right]^{\frac{38}{3b}} - 1\right] + \frac{5}{11}\left[\left[\frac{g^2(m_c)}{g^2(m_w)}\right]^{\frac{11}{3b}} - 1\right] \right\} \tag{8}$$

Account is taken in (8) of the fact that $b = 9$ in the region of momenta lower than m_c. If we choose for the numerical estimates of η_T

$$m = 0.7\ \Gamma\text{GeV},\ m_c = 2\ \text{GeV};\ m_w = 70\ \text{GeV};\ g^2(m)/4\pi = 1;\quad N = 4-8,$$

then

$$\eta_T \approx 0.76. \tag{9}$$

Our result differs quite substantially from the conclusions drawn in[1], according to which strong interactions at short distances enhance the contribution of the $V+A$ currents in nonleptonic interactions by approximately one order of magnitude. The reason for the discrepancy lies in the fact that in these papers they calculated only the anomalous dimensionality of the operator B_1, which is indeed large and positive. No account was taken, however, of the fact that the matrix elements of the operator B_1 are small. On the other hand, to calculate the admixture of the operator B_1 in T, the matrix elements of which are not small, it is necessary to carry out a much more complicated calculation of the two-loop diagrams. Our calculation has shown that the admixture at short distances of the operator B_1 to the operator T is small (the coefficient 1/19 in Eq. (8)).

We shall dwell in conclusion on an estimate of the matrix element of the operator T for the decay $K_S^0 \to 2\pi$. This estimate depends on the value of $\sin\phi$. The upper bound of $\sin\phi$ can be obtained from the estimate of the $K_L - K_S$ mass difference (for the formulas see, e.g., [3]). If we assume for the s-quark mass the value $m_s = 150$ MeV,[4] then

$$|\sin\phi| \lesssim 1/10\,. \tag{10}$$

The width of the decay $K_S^0 \to 2\pi$ will be estimated roughly by comparison with the width of the decay of the scalar resonance $\sigma \to 2\pi$. The strong interaction is described by the vertex $(g/2)\bar{\psi}\gamma_\mu t^a \psi b^a_\mu$ and the weak interaction by $G\sqrt{2}m_c(g/16\pi^2)\sin\theta_C \sin\phi\,\bar{\psi}\sigma_{\mu\nu}t^a\psi b^a_{\mu\nu}$. For the ratio of the widths we obtain naturally

$$\frac{\Gamma^{(T)}_{\text{weak}}}{\Gamma_{\text{strong}}} \sim \left(\frac{G m_c m\sqrt{2}}{8\pi^2}\sin\theta_C \sin\phi\right)^2$$

or

$$\Gamma^{(T)}_{\text{weak}}(K \to 2\pi) \sim 1/400\,\Gamma(K \to 2\pi)_{\text{exp}}\,, \tag{11}$$

if we assume $m = 0.7$ GeV, $\Gamma_{\text{strong}} \approx 300$ MeV, and take the bound (10) into account.

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