Gauge theories as spontaneous breaking theories

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(Submitted April 27, 1976)
Pis'ma Zh. Eksp. Teor. Fiz. **23**, No. 11, 661–664 (5 June 1976)

It is shown that any gauge theory is a spontaneous symmetry breaking theory with respect to a definite infinite-parametric group, and that the corresponding gauge field is the goldstonion that accompanies this spontaneous breaking.

PACS numbers: 11.30.Qc, 12.20.Hx

1. It is well known that there is a close analogy between gauge and Goldstone ields. This analogy is manifest, in particular, in the inhomogeneity of their group transformations.[1] We demonstrate in this article that this analogy is ιot accidental, and is due to the fact that any gauge theory can be regarded as ι theory of spontaneous breaking of a definite symmetry, and the corresponding gauge field can be regarded as the goldstonion accompanying this spontaneous breaking.

We develop and generalize in this paper the results of an article[2] in which t is shown that gravitation theory is a theory of spontaneously broken affine nd conformal symmetries, and the gauge field $h_{\mu\nu}$ (graviton) is a goldstonion. imilar in their conception are also the articles[3,4] in which the photon is inerpreted as a Goldstone field. We note that another approach to this problem, ased on introduction of additional dimensions of space, was proposed in a reent paper.[5]

2. We shall show first that the gauge transformation can be regarded as a ransformation from a certain group K with constant parameters and an infinite umber of generators. The commutation relations between these generators re uniquely determined by the algebra of the initial finite-parameter group G^0.

Confining ourselves to gauge functions that can be expanded in a Taylor seies in the vicinity of $x_\mu = 0$, we represent the element of a certain local-symıetry group $G^{l(x)}$ in the form

$$\exp\{ i a^\alpha(x) Q^\alpha\} \equiv \exp\{ i a^\alpha(0) Q^\alpha + \sum_{n\geqslant 1} a^\alpha_{\mu_1\ldots\mu_n} Q^\alpha_{\mu_1\ldots\mu_n}\}, \qquad (1)$$

here Q^α are the generators of a finite-parameter subgroup G^0 and

$$a^\alpha_{\mu_1\ldots\mu_n} \equiv \frac{1}{n!}\partial^x_{\mu_1}\ldots\partial^x_{\mu_n} a^\alpha(x)|_{x_\mu=0}, \qquad (2)$$

$$Q^\alpha_{\mu_1\ldots\mu_n} \equiv x_{\mu_1}\ldots x_{\mu_n} Q^\alpha .$$

he generators $Q^\alpha_\mu, \ldots Q^\alpha_{\mu_1\ldots\mu_n}, \ldots$ together with Q^α generate an infinite-parameter roup K. The commutation relations of these generators with one another and ith the generator of the 4-translations $P_\mu = -i\partial^x_\mu$ can be obtained by using the epresentation (2):

$$[Q^\alpha_{\mu_1\ldots\mu_k}, Q^\beta_{\mu_{k+1}\ldots\mu_n}] = iC^\gamma_{\alpha\beta} Q^\gamma_{\mu_1\ldots\mu_n}, \qquad (3)$$

$$[P_\rho, Q^\alpha_\mu] = -i\delta_{\rho\mu} Q^\alpha \,, \tag{4}$$

$$[P_\rho, Q^\alpha_{\mu_1 \dots \mu_n}] = -i\left(\delta_{\rho\mu_1} Q^\alpha_{\mu_2 \dots \mu_n} + \dots + \delta_{\rho\mu_n} Q^\alpha_{\mu_1 \dots \mu_{n-1}}\right). \tag{5}$$

$$(n \geqslant 2)$$

Here $C^\gamma_{\alpha\beta}$ are the structure constants of the subgroup G^0. The transformation properties of $Q^\alpha_{\mu_1 \dots \mu_n}$ relative to G^0 in the homogeneous Lorentz group L in the representation (2) are obvious. The group K together with the Poincare group $\mathcal{P}$ form the semi-direct product $\mathcal{K} = K \otimes \mathcal{P}$.

3. Our main result is that the gauge theory connected with the local group $G^l(x)$ can be obtained as a result of a Nambu-Goldstone realization[6,7] of the symmetry with respect to the group $\mathcal{K}$, with the subgroup $G^0 \otimes \mathcal{P}$ as the subgroup of the vacuum stability. The gauge field then turns out to be a goldstonion corresponding to the generator Q^α_μ.

The most direct and natural method of the Nambu-Goldstone realization of the symmetry is the method of nonlinear reactions, which has been developed in detail in[8,9]. According to[8,9], we should parametrize the factor-space $\mathcal{K}/G^0 \otimes L$ by the fields $b^\alpha_\mu(x), \cdots b^\alpha_{\mu_1 \dots \mu_n}(x), \cdots$ with the quantum numbers of the generators $Q^\alpha_\mu, \cdots Q^\alpha_{\mu_1 \dots \mu_n}, \cdots$ and regard the action of the group $\mathcal{K}$ in factor space as that of a group of left-hand shifts.

The Poincare group acts on the fields and on x_μ in standard manner, so that we are interested only in the action of the group K. The fields $b^\alpha_\mu(x), \cdots b^\alpha_{\mu_1 \dots \mu_n}(x), \cdots$ transform inhomogeneously:

$$Q^\alpha_\mu : \qquad \delta b^\alpha_\mu(x) = a^\alpha_\mu + O_1(b, x) (= a^\alpha_\mu - C^\alpha_{\beta\gamma} b^\beta_\mu(x)(x_\rho a^\gamma_\rho)\ ),$$

$$Q^\alpha_{\mu_1 \dots \mu_n} : \quad \delta b^\alpha_{\mu_1 \dots \mu_n}(x) = a^\alpha_{\mu_1 \dots \mu_n} + O_n(b, x).$$

These fields are therefore goldstonions. The other fields $\Psi(x)$ transform in accordance with the representations of the algebraic subgroup G^0 but with the function parameters $U^\alpha(x, k)$ (k is an element of the group K). If we confine ourselves to the algebraic subgroup of the field $\Psi(x)$, then $b^\alpha_\mu(x), \cdots b^\alpha_{\mu_1 \dots \mu_n}$ transforms in accordance with the corresponding representations of this subgroup with constant parameters.

The invariant Lagrangians are constructed in standard fashions from the fields and their covariant derivatives.[8,9] Owing to the specific structure of the group $\mathcal{K}$, the covariant derivatives turn out to be polynomials in the goldstonions and can be easily obtained in explicit form. Using the general prescriptions,[8,9] we obtain

$$\nabla_\rho b^\alpha_\mu(x) = \partial_\rho b^\alpha_\mu(x) + b^\alpha_{\rho\mu}(x) - \frac{1}{2} C^\alpha_{\beta\gamma} b^\beta_\rho(x) b^\gamma_\mu(x), \tag{6}$$

$$D_\rho \Psi(x) = (\partial_\rho + i b^a_\rho(x) Q^a)\Psi(x). \qquad (7)$$

t is easy to duplicate also the remaining covariant derivatives.

It is important that the symmetrical and antisymmetrical parts of $\nabla_\rho b^\alpha_\mu$, viz.,

$$\nabla_{\{\rho} b^a_{\mu\}} = \partial_\rho b^a_\mu(x) + \partial_\mu b^a_\rho(x) + 2 b^a_{\rho\mu}(x), \qquad (8)$$

$$\nabla_{[\rho} b^a_{\mu]} = \partial_\rho b^a_\mu(x) - \partial_\mu b^a_\rho(x) - C^a_{\beta\gamma} b^\beta_\rho(x) b^\gamma_\mu(x) \qquad (9)$$

ransform independently of each other. In this case $\nabla_{[\rho} b^\alpha_{\mu]}$, just as $D_\rho\Psi$, does ıot contain goldstonions with ≥ 2 tensor indices. $\nabla_{[\rho} b^\alpha_{\mu]}$ coincides in structure vith the covariant Yang-Mills rotor. Thus, to construct the invariant Lagran- ;ians it suffices to have at our disposal $\nabla_{[\rho} b^\alpha_{\mu]}$, $D_\rho\Psi$ and Ψ

$$\mathcal{L}^{inv}_{(\mathcal{K})} = \mathcal{L}^{inv}_{(G^0 \otimes L)} (D_\rho \Psi, \Psi, \nabla_{[\rho} b^a_{\mu]}).$$

The connections of the goldstonion $b^\alpha_\mu(x)$ with itself and with the field $\Psi(x)$ are dentical with the connections of the Yang-Mills gauge field, and it is this vhich allows us to identify $b^\alpha_\mu(x)$ with this gauge field.

The goldstonions $b^\alpha_{\mu_1 \ldots \mu_n}$ $(n \geq 2)$ turn out in final analysis to be inessential. They can be eliminated by expressing in terms of the vector goldstonion and ts derivatives with the aid of the inverse Higgs effect[10] (by equating to zero he symmetrical parts of the covariant derivatives of the goldstonions).

We have considered above only the case of internal symmetries. The entire nalysis, however, can be easily transferred also to the case when the initial ;roup G^0 determines the space-time symmetry.

4. Thus, any gauge theory is a spontaneous symmetry-breaking theory.[1)] Consequently, spontaneously broken symmetries constitute a more profound nd more general concept than gauge theories. A discussion of the ensuing onsequences will be presented in a detailed article.

[1)]The statement[11] that there exist no goldstonions with spins $> 1/2$ is not applicable to gauge fields, since it makes essential use of the definiteness of the metric of the space of state and the explicit relativistic invariance—assumptions that cannot be satisfied simultaneously in any gauge theory.

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