Cerenkov production pions in collisions of relativistic nucleons with nuclei

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One of the possible mechanisms of pion production by collision of relativistic nucleons with nuclei, analogous to the Cerenkov effect in electrodynamics, is considered. Consequences that make it possible to observe this process experimentally are analyzed.

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Experimental data connected with coherent production of pions in collisions between relativistic particles and nuclei have been reported recently.[1] The theoretical interpretation of the obtained spectra on the basis of the mechanism of pion production in the nucleon-nucleon interaction process meets with certain difficulties. In particular, it is impossible to explain why the obtained ion spectra contain none of the resonances that usually appear in the case of nucleon-nucleon interactions.[1]

One of the possible mechanisms of pion production in collisions between a nucleon and a nucleus may be a process analogous to the Cerenkov effect, which is well known in electrodynamics.[2] We shall consider here this pion-production mechanism. In vacuum, a free nucleon cannot emit a pion, for then the energy and momentum conservation laws are not satisfied simultaneously. In nuclear matter, the momentum of the pion is altered by the strong interaction. The ensuing changes can be described phenomenologically by introducing a refractive index for the pion wave, so that the energy and momentum conservation laws can be simultaneously satisfied for the emission of a pion by a nucleon in nuclear matter.

Let us consider in detail the case of pseudoscalar interaction of a pion with a nucleon. The Hamiltonian of this interaction then takes the form[3]

$$H = iG\bar{\psi}\gamma_5\tau\psi\phi\,, \tag{1}$$

where G is the effective constant (vertex) of the interaction of the nucleon with the pion in the medium, $\bar{\psi}$ and ψ constitute the nucleon field, ϕ is the meson field, τ is the isotopic-spin operator, and γ_5 is a Dirac matrix. We shall assume that the wave functions of the relativistic nucleons $\psi_{q'}$ and ψ_q are Dirac plane waves with 4-momenta q' and q, respectively. Then, using (1) we can obtain an expression for the probability of the transition of the nucleon from the state q to the state q' with emission of a pion with 4-momentum $k=(\epsilon_\pi, pn)$, where $n(\epsilon_\pi)$ is the refractive index of the nuclear matter for pions with energy $\epsilon_\pi = \sqrt{m_\pi^2 + \mathbf{p}^2}$ $(\hbar = c = 1)$

$$dw_{+,0} = \frac{n^3(\epsilon_\pi)\,d\mathbf{p}\,d\mathbf{q}}{32\pi^2\epsilon_\pi\epsilon'\epsilon}\,\delta^4(q - q' - k)\,|G|^2 A_{+(0)}[(qq') - m_p^2], \tag{2}$$

where $A_+ = 2$ if charged $\pi^\pm$ pions are emitted and $A_0 = 1$ for neutral pions, $q(\epsilon, \mathbf{q})$, $q' = (\epsilon', \mathbf{q}')$ are the 4-momenta of the nucleons before and after the pion emission, and m_p is the nucleon mass. To determine the condition for the existence of such a process, we investigate the laws of energy and momentum conservation (we neglect the recoil of the nucleus)

$$\epsilon = \epsilon' + \epsilon_\pi, \qquad \mathbf{q} = \mathbf{q}' + n\mathbf{p}. \tag{3}$$

Solving the system (3), we obtain an expression for the cosine of the angle between the momentum $\mathbf{q}$ of the incident nucleon and the momentum of the emitted pion in the nuclear medium:

$$\cos\theta = \frac{\epsilon_\pi}{nv(\epsilon_\pi^2 - m_\pi^2)^{1/2}}\left(1 + \frac{\epsilon_\pi(n^2 - 1)}{2\epsilon} - \frac{n^2 m_\pi^2}{2\epsilon\epsilon_\pi}\right), \tag{4}$$

where v is the velocity of the incident nucleon. The condition for the existence of radiation, just as in the usual Cerenkov effect, is $|\cos\theta| \leq 1$. From this condition we easily obtain the threshold value of the pion energy at a given velocity of the incident nucleon $(\epsilon_\pi/\epsilon \ll 1)$:

$$\left(\frac{\epsilon_\pi}{m_\pi}\right)^2 \geqslant \frac{(vn)^2}{(vn)^2 - 1}\,. \tag{5}$$

Integrating expression (2) with respect to $d\mathbf{q}'$ and with respect to the direction of the pion momentum in the medium, we obtain for the spectrum of the emitted pions the expression $(v \approx 1)$

$$w_{+(0)} = \frac{|G|^2}{16\pi}A_{+(0)}\frac{n^2(\epsilon_\pi)\,d\epsilon_\pi}{\epsilon|\mathbf{p}|}[\epsilon_\pi^2(n^2 - 1) - n^2 m_\pi^2]\,. \tag{6}$$

We can obtain analogously an expression for the spectrum of the emitted pions in the case of the pseudovector variant of the interaction $(v \approx 1)$:

$$w_{+,(0)} = \frac{|F|^2}{8\pi}A_{+(0)}\frac{m_p^2}{m_\pi^2}\,\frac{n^2 d\epsilon_\pi}{\epsilon|\mathbf{p}|}[\epsilon_\pi^2(n^2 - 1) - n^2 m_\pi^2], \tag{7}$$

where F is the effective constant (vertex) of the pseudovector variant of the interaction in nuclear matter. Since the refractive index $n(\epsilon_\pi)$ is determined for low pion energies mainly by the amplitude of the pion-nucleon scattering in the region of the Δ_{33} resonance, it follows that $n^2(\epsilon_\pi) > 1$. The numerical value of the refractive index can be obtained by using the parameters of the Kislinger optical potential for the pions.[4]

In the pion kinetic-energy range from 0 to 100 MeV, n^2 takes on values from to 11. It follows therefore that the energy threshold for the production of pions by a nucleon in a medium with $v \sim 1$, as follows from (5), is of the order of $(\epsilon_\pi/m_\pi)^2 \sim 4/3$. The maximum of the spectral distribution of the emitted pions should lie on the left of the Δ_{33} maximum, at the point where the refractive index is maximal. Thus, an additional indication of the role of the Čerenkov mechanism of pion emission in collision of relativistic nucleons with nuclei can be obtained by measuring the angular distribution of these pions. The angle θ' at which the pion is emitted in vacuum from the nucleus is determined from the boundary condition on the surface of the nucleon:

$$n \sin(\theta - \theta_n) = \sin(\theta' - \theta_n) \quad (8)$$

θ_n is the angle between the normal to the surface of the nucleus and the direction of the incident-nucleon momentum. Near the pion production threshold we have $\theta \approx 0$, and then we get from (8)

$$\sin\theta' = \sin\theta_n (n \cos\theta_n \pm \sqrt{1 - n^2 \sin^2\theta_n}). \quad (9)$$

It follows from (9) that near the production threshold the value of the angle θ' varies in the interval $0 \leq \theta' \leq \cos^{-1}(1/n)$, or for $n \approx 2$ we have $0 \leq \theta' \leq \pi/3$.

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