Contribution to the theory of trigger radiation

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A theory is constructed for the generation of monochromatic radiation when the magnetosphere is probed by whistlers of sufficiently large amplitude. The investigated generation mechanism is based on velocity bunching of the resonant particles in the field of the main wave. Radiation with time-varying frequency is produced when the resultant bunches move in the geomagnetic field.

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Experiments on the propagation whistlers in a geomagnetic field have revealed the generation of a trigger, namely, at sufficiently large transmitter power, the main signal received at the conjugate point was accompanied by a monochromatic radiation with a frequency that varied with time. [1,2]

In this article we develop a theory for the mechanism that produces this radiation. The main wave leads to modulation of the plasma distribution function in the region of the resonant velocities $v^{\rm res} \approx -\omega_{H0}/k_0$ (k_0 is the wave number of the main wave and ω_{H0} is the cyclotron frequency corresponding to the magnetic field at the resonance). The bunches formed by such a modulation produce in the plasma a response at the wavelength $\lambda = 2\pi v^{\rm res}/\omega_H(z)$ (z is the coordinate of the bunch), which is received at the conjugate point as monochromatic radiation with time-varying frequency. 1)

The considered trigger-generation mechanism is analogous to a considerable degree to the linear echo[5] and is due to the fact that in an inhomogeneous magnetic field there exist resonant velocities for which the phase mixing is slow enough.

To realize this phenomenon it is necessary to have a sufficiently rapid variation of the geomagnetic field, at which the particle ceases to be resonant with the wave before the onset of the phase mixing due to the oscillations of the resonant particles:

$$v^{\rm res}\frac{d\omega_H}{dz} \gg \Omega_T^2\,,\ \text{i.e.,}\ \frac{1}{\sqrt{k_0 L}} \gg \frac{\Omega_T}{\omega_H}\,. \tag{1}$$

Here $\Omega_T = \sqrt{\Omega_H k_0 v_T}$ is the characteristic frequency of the phase oscillations of the trapped particles, Ω_H is the electron cyclotron frequency in the magnetic field of the principal wave, and L is the radius of the force line. To find the distribution function of the resonant particles it suffices in this case to use a linearized kinetic equation. The modulation of the distribution function is produced by the main wave, which constitutes a packet with a sufficiently steep rising front:

$$\mathcal{H} = \mathcal{H}_0 e^{i\left(\int_0^z k_0\,dz - \omega_0 t\right)}\,\sigma\!\left(t - \int \frac{dz}{d\omega_0/dk}\right) \tag{2}$$

where $\sigma(x)$ is the unit step function.

The magnetic field of the response at the frequency ω is then determined by the formula

$$\mathcal{H}_\omega = -\mathcal{H}_0 \frac{e^{\pi i/2}}{2n_0 \frac{d\omega_0}{dk} k_0} \omega_H(z_1)\omega_H(z_2) \frac{\omega_H(z_2)-\omega}{\omega-\omega_0} e^{i\int_0^z k_\omega dz - i\omega t}$$

$$\times \int d\epsilon\, dv_\perp \frac{v_\perp}{m v_z^3}\left(\frac{\partial f_0}{\partial v_\perp} - \frac{v_\perp}{v_z}\frac{\partial f_0}{\partial v_z}\right)\bigg|_{z=z_1} \frac{e^{-i\Psi(z_1,z_2,\epsilon)}}{\sqrt{\frac{d^2\Psi}{dz_1^2}\frac{d^2\Psi}{dz_2^2}}}. \quad (3)$$

The phase shift is

$$\Psi(z_1,z_2,\epsilon) = \int_0^{z_1} k_0\, dz + \int_{z_2}^{0} k_\omega dz - \int_{z_1}^{z_2} \frac{\omega_H - \omega_s(z_2)}{v_z(z,\epsilon)} dz + (\omega_s(z_2)-\omega_0)\int_{-\infty}^{z_1} \frac{dz}{d\omega_0/dk},$$

$v_z(z,\epsilon)$ is determined from the relation $v_z = \sqrt{(2/m)(\epsilon-\mu H)}$, ϵ is the energy and μ is the magnetic moment of the particle. The coordinates of the points z_1 and z_2 are determined from the resonance conditions

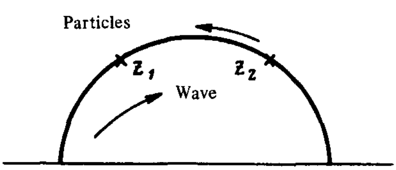
$$k_0(z_1) + \frac{\omega_H(z_1)-\omega}{v_z(z_1,\epsilon)} + \frac{\omega-\omega_0}{d\omega_0/dk} = 0, \quad (4)$$

$$k_\omega(z_2) + \frac{\omega_H(z_2)-\omega_s(z_2)}{v_z(z_2,\epsilon)} - \frac{d\omega_s}{dz_2}\left(\int_{z_1}^{z_2}\frac{dz}{v_z(z,\epsilon)} + \int_{-\infty}^{z_1}\frac{dz}{d\omega_0/dk}\right) = 0,$$

z_1 is the point of the resonance between the main wave and the particles with velocity v_z; these particles move in a direction opposite to the wave propagation and produce at the point z_2 a response at the frequency ω (see the figure); $\omega_s(z_2)$ is the frequency of the "whistler," which at $z = z_2$ is at resonance with the particles with velocities v_z:

$$\omega_s(z_2) = c^2/v_z^2 \quad \omega_H^3(z_2)/\omega_p^2(z_2);$$

the second of the resonance conditions (4) denotes that $\omega_s(z_2) \approx \omega$. At a sufficiently large distance between the points z_1 and z_2, the response frequency ω can differ noticeably from the frequency ω_0 of the main wave. The amplitude of the response is appreciable in this case only when the integral with respect to ϵ in (3) contains the extremum points $|d\Psi/d\epsilon^*| = 0$ of the phase $\Psi(z_1,z_2,\epsilon)$,



and consequently according to (3), the following condition is satisfied at $\epsilon = \epsilon^*$:

$$\int_{z_1}^{z_2} (\omega_H - \omega_s) \frac{dz'}{v_z^2(z', \epsilon^*)} + \frac{\partial \omega_s}{\partial v_z}\left(\int_{z_1}^{z_2} \frac{dz}{v_z(z, \epsilon^*)} + \int_{-\infty}^{z_1} \frac{dz}{d\omega_0/dk}\right)$$

$$+ (\omega_s - \omega)\frac{dz_1}{dv_z}\left(\frac{1}{v_z(z_1, \epsilon^*)} - \frac{1}{d\omega_0/dk}\right) = 0 . \qquad (5$$

The phase extremum condition $d\Psi/d\epsilon^* = 0$ is a fundamental one in the linear-echo theory and corresponds to bunching of the resonant particles at velocities $v_z^* = v_z(\epsilon^*, z)$ in the interval $\Delta v_z \approx v_z^*/\sqrt{k_0 L}$.

Calculating the integral with respect to ϵ by the saddle-point method and changing from the Fourier transform $\mathcal{H}_\omega$ to the field $\mathcal{H}(t, z)$, we obtain

$$\mathcal{H}(t, z) = -\pi \frac{\omega_H(z_2)\omega_H(z_1)}{n_0 m k_0 v_z^{*3}} \frac{\omega_H(z_2) - \omega_T}{\omega_T - \omega_0} \frac{e^{\pi i/2}}{d\omega_T/dk}$$

$$\times \frac{\mathcal{H}_0}{\sqrt{\frac{d^2\Psi}{dz_1^2}\frac{d^2\Psi}{dz_2^2}}} \int d v_\perp v_\perp \left(\frac{\partial f_0}{\partial v_\perp} - \frac{v_\perp}{v_z^*}\frac{\partial f_0}{\partial v_z^*}\right)\Bigg|_{z = z_1} \frac{e^{-i\Gamma(\omega = \omega_T)}}{\sqrt{\frac{d^2\Psi}{d\epsilon^{*2}}\frac{d^2\Gamma}{d\omega_T^2}}} . \qquad (6$$

The integration with respect to $v_\perp$ is carried out here between the limits

$$v_\perp^2 \leqslant v_z^2(\epsilon^*, z_1)\frac{H(z_1)}{H(z_2) - H(z_1)} ;$$

$$\Gamma = \omega t - \int_0^z k_\omega dz + \Psi(z_1, z_2, \epsilon^*).$$

The trigger frequency $\omega_T(t, z)$ as a function of the instant of time t and of the coordinate z of the observation point is determined from the condition

$$t = \int_{z_2}^{z} \frac{dz}{d\omega/dk} + \int_{z_1}^{z_2} \frac{dz'}{v_z(z', \epsilon^*)} + \int^{z_1} \frac{dz'}{d\omega_0/dk} . \qquad (7$$

With increasing time, the interval between the point z_1 where the bunch is produced and the point z_2 where the response appears increases, and the advance into the region of larger energies of the geomagnetic field is accompanied by an increase in the trigger frequency

$$\omega_T \approx \frac{c^2}{v_z^2(z_2, \epsilon^*)} \frac{\omega_H^3(z_2)}{\omega_p^2(z_2)} .$$

In the mechanism considered here, the initial trigger amplitude $\mathcal{H} \sim \mathcal{H}_0(n'/n_0)$

$\times [\omega_H/(\omega_T - \omega_0)]$ (n' is the density of the resonant particles and n_0 is the plasma density) greatly exceeds the thermal-noise level, and when account is taken of the amplification accompanying the inward propagation from the point z_2 to the observation point z, the amplitude of the signal can turn out to be comparable with the amplitude of the main wave, or may even exceed it.

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1) We note that a phenomenological theory of triggers was constructed earlier in[3], where the existence of particle bunches resulting from cyclotron interaction of the particles with the wave was postulated. Another approach, based on the assumption that the trigger is caused by satellite instability on the trapped particles, was developed in[4].

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