HF diamagnetism and three-dimension cyclotron solitons in a plasma

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Cyclotron oscillations at double, triple, etc. the electron cyclotron frequency can produce non-spreading three-dimensional packets—solitons. They can be fed by a beam of electrons or, in the case of HF heating, by an electromagnetic wave. In an unstable plasma, solitons have a certain margin of stability. The x-ray bursts sometimes observed in tokomaks can be attributed to the collapse of a large number of excitons.

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It is known from experiment that cyclotron oscillations have a strong influence on the conductivity and heating of a plasma in a magnetic trap. However, it follows from an analysis in the linear approximation that they can be easily scattered and be radiated from the surface of the plasma, owing to the dispersion spreading of the wave packet. We shall show that this spreading is impeded by a nonlinear effect, namely the formation of a magnetic well in the region of existence of the wave packet. A standing three-dimensional stationary packet, or three-dimensional soliton, is then produced.

We consider in this paper a three-dimensional soliton at double the electron cyclotron frequency. We obtain the average diamagnetic current produced by the high-frequency (HF) pressure in the soliton. We assume that the soliton has cylindrical symmetry and that its length (dimension along the magnetic field) is much larger than the thickness. Then the largest contribution to this current is made by the following part of the oscillating component of the electron distribution function:

$$f_1 = \frac{iev_\perp e^{-ia}}{4m\omega_H(\omega - 2\omega_H)^2} v_z \frac{\partial}{\partial z}\left(e^{-ia} + \frac{iv_\perp}{\omega_H}\frac{\partial}{\partial z_\perp}\right)\Delta_\perp\phi \frac{\partial f_0}{\partial v_\perp}. \tag{1}$$

Here ω_H is the cyclotron frequency of the electrons, $\phi(r_\perp, z)$ is the electric potential, ω is the soliton frequency, α is the angle in velocity space between the vectors $\mathbf{v}_\perp$ and $\mathbf{r}_\perp$, and the z axis is directed along the magnetic field. We assume that the Larmor radius is small. Substituting (1) in the quasilinear equation averaged over the soliton frequency, we obtain the correction to the averaged electron distribution function:

$$\delta f_0 = \frac{e^2}{16m^2}\frac{[\mathbf{v}_\perp \nabla_\perp]_z}{\omega_H^3(\omega - 2\omega_H)^2}|\Delta_\perp\phi|^2 \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} v_\perp^3 \frac{\partial f_0}{\partial v_\perp}. \tag{2}$$

δf_0 produces not a space charge but a closed current that leads to a diamagnetic decrease of the magnetic field. The corresponding change of the cyclotron frequency can be easily obtained from Maxwell's equation and the expression obtained from (2) for the current. It is equal to

$$\delta\omega_H = -\omega_p\beta^2 \frac{v_T^2}{(\omega - 2\omega_H)^2} \frac{|\Delta_\perp\phi|^2}{H^2} \quad ; \quad \beta = \omega_p/2\omega_H \, . \tag{3}$$

Here v_T is the thermal velocity of the electrons and H is the constant magnetic field.

The dispersion equation of the considered oscillation mode is of the form

$$\omega = 2\omega_H[1 + (\beta k_\perp r_H)^2/2 + k_z^2 (2k_\perp\beta)^{-2}] \, . \tag{4}$$

Here r_H is the Larmor radius of the electrons. It is assumed that $k_\perp r_H \ll 1$ and $k_z \ll k_\perp$. From (4) we obtain with allowance for the nonlinear correction (3) an equation for ϕ in the stationary case. Introducing the notation $\omega - 2\omega_H = -\omega_H A^2$, we have

$$\Delta_\perp(A^2\phi - \beta^2 r_H^2 \Delta_\perp\phi) + \frac{1}{2\beta^2}\frac{\partial^2\phi}{\partial z^2} = \beta^2 r_H^2 (A^2 H)^{-2} |\Delta_\perp\phi|^2 \Delta_\perp\phi \, , \tag{5}$$

the solution of which relative to $\Delta_\perp\phi$ is finite everywhere and tends to zero at infinity, namely,

$$\Delta_\perp\phi = A^3 \frac{H}{\beta r_H} F\left(A\frac{r_\perp}{\beta r_H} \, , \, \sqrt{2}A^2\frac{z}{r_H}\right) , \tag{6}$$

where A is the dimensionless amplitude of the soliton $A \ll 1$ in our approximation. $F(\rho, \zeta)$ is a function satisfying the equation

$$\Delta_\rho(F - \Delta_\rho F) + \frac{\partial^2 F}{\partial\zeta^2} = \Delta_\rho F^3$$

$$\Delta_\rho \equiv \frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho} \quad ; \quad F(\rho, \zeta) = -F(\rho, -\zeta). \tag{7}$$

Equation (7) was solved with a computer. It was found that F differs from zero in a region with dimensions on the order of unity, where $|F| \sim 1$. Therefore the electric field in the soliton has the following parameters: $|E_\perp| \sim A^2 H$, the soliton dimension $l_z \sim r_H/A^2$ along the magnetic field, and the dimension $l \sim (\omega_p/\omega_H)(r_H/A)$ across the magnetic field. The total soliton energy is

$$W = \frac{1}{8\pi}\int \omega \frac{\partial\zeta}{\partial\omega} E^2 d\mathbf{r} = 1.26\left(\frac{\omega_p}{\omega_H}\right)^2\left(\frac{H}{A}\right)^2 r_H^3 \quad . \tag{8}$$

Inasmuch as the soliton energy increases together with the volume, such a soliton can collapse. [1] Expanding solutions exist also simultaneously with the collapsing solutions. In the expanding-solution regime, the total energy of the wave packet increases according to (8), and therefore such a solution cannot be realized in a stable plasma. Such a solution is possible, however, in the presence of an instability that transfers energy to the packet. The packet expands and its energy increases, but in such a way that the energy density in the packet decreases and the packet becomes more "linear." With decreasing nonlinearity, the expansion of the packet can be easily hindered by dissipation, by loss of resonance with the instability, etc. Then the packet goes over into a stationary state. In this state, the packet has a certain stability margin with respect to collapse. Examples of such stationary packets were considered in[2,3]. The energy can be pumped into the soliton from the higher-frequency oscillations via decay or induced scattering. A three-dimensional soliton can be amplified by beams even if it is immobile. As is well known, an immobile one-dimensional soliton is not amplified by a beam.[4] The reason is that in the three-dimensional soliton, in contrast to the one-dimensional one, the energy distribution over the phase velocities has a maximum.

In the case when the pumping is by a beam of fast electrons, the energy accumulated in the solitons and the number of solitons increase. The solitons then begin to interact with one another and this leads to loss of stability and to collapse of a large number of solitons. Collapse deepens the magnetic well, and the electrons are trapped in it and acquire transverse-motion energy at the expense of the soliton energy.

It is probable that the quasiperiodic bursts of the number of electrons with large transverse energy, observed in tokomak plasma in the presence of runaway electrons[5] can be attributed to collapse of cyclotron solitons.

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