Behavior of the wave function at short distances and blocking of quarks

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The relative motion of quarks is described in a relativistic coordinate space, the transition to which is realized with the aid of an expansion of the wave functio on the Lorentz group. It is shown that inclusion of an additional series of unita representations in this expansion leads in natural fashion to a potential that trap the quarks.

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In recently proposed models, the forces binding the quarks increase with increasing relative distance, leading to trapping of the quarks inside the particle and to the impossibility of observing them in the free state.

We have investigated the problem of quark trapping within the framework c the quasi-potential approach,[1] using for this purpose Kadyshevskiĭ's equation.[2] In the quasipotential equations, in contrast to the Bethe–Salpeter equa tion, the momenta of all the particles lie on the mass shell. It is therefore convenient to change over here to the relativistic configuration representatio (RCR) introduced within the framework of Kadyshevskiĭ's approach in [3]. Th RCR differs from the nonrelativistic coordinate representation in that in this case the Shapiro transformation [4] is used in place of the Fourier transformation. The Shapiro transformation is an expansion in the matrix element of the principal series (PS) of unitary irreducible representations of the Lorentz group (LG)—the group of motion of the mass-shell hyperboloid $p_0^2-p-M^2$, and for the wave function of the relative motion of two quarks it takes the following form, in the notation of [3],

$$\Psi(\mathbf{p}) = \int \xi(\mathbf{p},\mathbf{r})\Psi(\mathbf{r})\,d\mathbf{r}; \qquad \xi(\mathbf{p},\mathbf{r}) = \left(\frac{p_0-\mathbf{p}\mathbf{n}}{M}\right)^{-1-irM}, \quad \mathbf{n}^2 = 1.$$

Here p is the momentum of the quark in the c.m.s., and $\mathbf{p}_1=-\mathbf{p}_2=\mathbf{p}$. It was proposed in [3] to regard the parameter r, which determines the eigenvalues X^2 of the Casimir operator of the Lorentz group (LG) $\hat{C}=(1/4)M_{\mu\nu}M^{\mu\nu}$ ($M_{\mu\nu}$ a the generators of the group)

$$\hat{C}\,\xi(\mathbf{p},\mathbf{r}) = X^2\xi(\mathbf{p},\mathbf{r}); \quad X^2 = \frac{1}{M^2} + r^2 \quad (0 \leqslant r < \infty)$$

as a relativistic generalization of the relative coordinate. In the quasipotenti equation, written in the RCR, the role of the potentials is played by the form of the Feynman propagators. Thus, the propagator $1/(p-k)^2$, which describ the exchange of a zero-mass gluon, corresponds to the relativistic attractio potential

$$V(r) = -\frac{1}{4\pi r}\,\mathrm{cth}\,\pi r M \,.$$

virtue of (2) and the equality

$$< r_0^2 > \equiv 6 \frac{\partial F(t)}{\partial t}\Big|_{t=0} = \{ \hat{C} F(t) \}\Big|_{t=0}$$

e coordinate r describes distances that exceed the Compton wavelength.[5]

According to [5], a transition to distances shorter than the Compton wave-ngth, $X^2 < 1/M^2$, can be attained by including in the expansion of the wave nction a supplementary series (SS) characterized by the following values of e LG Casimir operator $\hat{C} \to X^2 = 1/M^2 - \rho^2$, where $0 \le \rho \le 1/M$. The coordinate s reckoned from the boundary of the sphere $X^2 = 1/M^2$ towards the center, that the value $\rho = 1/M$ corresponds to the origin $X^2 = 0$.

The analog of the plane waves of the principal series $\xi(\mathbf{p}, \mathbf{r})$ for the supple-entary series are the functions $\zeta(\mathbf{p}, \vec{\rho}) = [(p_0 - \mathbf{p}\cdot\mathbf{n})/M]^{-1-\rho M}$, which can be rmally obtained from $\zeta(\mathbf{p}, \mathbf{r})$ by the substitution $r \to i\rho$.[7] The expansion of), with allowance for the supplementary series, takes for the states with 0 the form

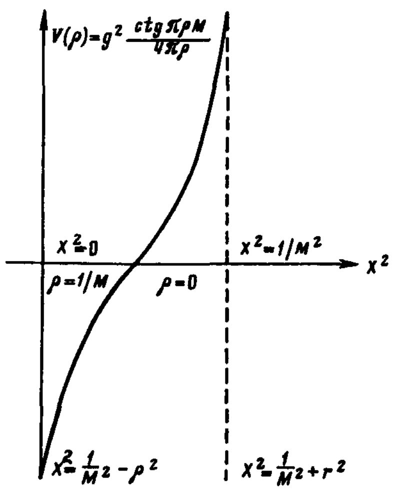
$$\Psi_{l=0}(\mathbf{p}) = 4\pi \int_0^\infty \frac{\sin rM\chi}{rM\,\mathrm{sh}\,\chi} \Psi(r) r^2 dr + 4\pi \int_0^{1/M} \frac{\mathrm{sh}\,\rho M\chi}{\rho M\,\mathrm{sh}\,\chi} \Psi(\vec{\rho}) \rho^2 d\vec{\rho}. \qquad (4)$$

We consider now the analog of the relativistic Coulomb potential at distances orter than $1/M$. Changing over in (3) to the supplementary series by making e substitution $r \to i\rho$, we obtain the potential (see the figure)

$$V(\rho) = \frac{1}{4\pi\rho} \mathrm{ctg}\,\pi\rho M; \quad 0 < \rho \leqslant 1/M, \qquad (5)$$

ich traps the quarks inside a sphere with $R^2 = X^2 = 1/M^2$. The free Hamil-nian operator $\hat{H}_0$ for the plane waves of the supplementary series is $\zeta(\mathbf{p}, \vec{\rho})$ $E_p \zeta(\mathbf{p}, \vec{\rho})$; $2E_p = 2M \cosh\chi = 2\sqrt{M^2 + \mathbf{p}^2}$;

$$\hat{H}_0 = 2M \mathrm{ch} \frac{1}{M} \frac{\partial}{\partial \rho} + \frac{2M}{\rho} \mathrm{sh} \frac{1}{M} \frac{\partial}{\partial \rho} - \frac{\Delta_{\theta,\phi}}{\rho^2}, \qquad (6)$$



just as in the case of the principal series,[3] is a differential-difference operator. The solution of the quasi-potential equation with potential (5)

$$(\hat{H}_0 + V(\rho))\Psi_q(\vec{\rho}) = 2E_q\Psi_q(\vec{\rho}) \quad (7)$$

in the region $0 \leq X^2 \leq 1/(2M)^2$, where $\cot\pi\rho M < 0$ and $M_{\text{bind}} \equiv 2E_q = 2M\cos x$ takes the following form for states with $l = 0$:

$$\Psi_{q,l=0}(\rho) = (e^{-ix}\sin x)e^{-ix\rho}\rho\exp\left[x\frac{\text{ctg}\,\pi\rho M}{2\sin x}\right]{}_2F_1\left(1+\rho M,\ 1+i\frac{\text{ctg}\,\pi\rho M}{2\sin x};\ 2;\ 2ie^{-ix}\sin x\right). \quad (8)$$

The function $\cot\pi\rho M$ in (5) is constant with respect to the operation of the finite difference differentiation (see [3]), by virtue of which it can play the role of the effective coupling constant in (7). The requirement that the solution be regular at $X^2 = 0$ ($\rho = 1/M$) leads to the quantization condition $\sin 2x = x$, which determine two energy levels. One with $M_{\text{bind}} = 2E_q = 1.38M$, and the other with $M_{\text{bind}} = 2E_q = 2M$. In the region $1/(2M)^2 \leq X^2 < 1/M^2$, where $\cot\pi\rho M > 0$ and $2E_q = 2M\cosh\chi \geq 2M$, the wave function is obtained from (7) by the substitution $x \to -i\chi$. The requirement of regularity of the point $X^2 = (1/M^2)(\rho = 0)$ leads to the condition $2\sinh\chi\, e^{-\chi} = \chi$, which determines one more level with $M_{\text{bind}} = 2E_q = 2.98M$.

Thus, in a quark-antiquark system situated in the field of the potential (5) in a state with $l = 0$, there can exist three energy levels or three excited states of one particle. For example, at a quark mass $M_q = 555$ MeV we obtain three ρ-meson states: $M_\rho = 765$ MeV, $M_{\rho'} = 1100$ MeV; and $M_{\rho''} = 1645$ MeV, which are close to the experimental values of the ρ-meson masses.

The supplementary-series functions $\xi(p, \vec{\rho})$ are not square-integrable.[7] This makes it necessary to include a regularizing kernel $K[(p-k)^2]$ in momentum space in the definition of the scalar products of the wave functions (8) describing distances shorter than $1/M$, that is, $(\Psi_1, \Psi_2) = \int\Psi_1(\mathbf{p})K[(p-k)^2]\Psi_2(\mathbf{k}) \times (d^3\mathbf{p}/p_0)(d^3\mathbf{k}/k_0)$. Questions of normalization of the wave functions (8) and of the description of the spectrum of the mesons and Ψ particles in our model with a quark-trapping potential (5) will be dealt with in a forthcoming article.

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