RESONANT SELF-TRAPPING OF HYPERSOUND

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A strong hypersonic wave of frequency 9.4 GHz propagating through an Al_2O_3 crystal containing paramagnetic Ni^{3+} ions that have a strong elec-ron-phonon bond was observed to be self-trapped in the region of acoustic parametric resonance.

1. Self-action effects, including self-focusing and self-trapping of a wave, are the consequence of the dependence of the propagation conditions (velocity and absorption coefficient) in a nonlinear medium on the wave intensity. These effects in nonlinear optics, as is well known, have been the subject of numerous recent investigations. In acoustics, owing to the relatively weak elastic nonlinearity of ordinary media, and also because of the appreciable difficulties when it comes to obtaining hypersound of very high intensity, such self-action of the wave has not yet been realized. From this point of view, interest attaches to acoustic paramagnetic resonance, which, owing to the nonlinearity of the characteristics of the paramagnetic system in the resonant region, makes it possible to realize self-trapping or self-focusing of an acoustic wave propagating in a medium with paramagnetic centers.

2. If a hypersound beam has a round cross section of radius a as it enters a resonant medium, its stationary propagation in the medium is described by the system of equations

$$\frac{\partial R}{\partial x} + \frac{1}{a^2}\frac{\partial R}{\partial r}\frac{\partial S}{\partial r} + \frac{R}{2a^2}\left[\frac{1}{r}\frac{\partial^2 S}{\partial r} + \frac{\partial^2 S}{\partial r^2}\right] + I_R(R) = 0, \tag{1}$$

$$\frac{\partial S}{\partial x} + \frac{1}{2a^2}\left(\frac{\partial S}{\partial r}\right)^2 = \frac{1}{2Rk^2a^2}\left[\frac{1}{r}\frac{\partial R}{\partial r} + \frac{\partial^2 R}{\partial r^2}\right] + I_S(R), \tag{2}$$

where R and S are the amplitude and the eikonal of the hypersonic wave and are slowly varying functions of the coordinates x and r in the longitudinal and transverse directions, k is the wave vector, and $I_R(R)$ and $I_S(R)$ are nonlinear functions of R and are expressed in terms of the characteristics of the paramagnetic system. If this system has an effective spin S' = 1/2 and the inhomogeneously broadened resonance line has a Lorentz shape, then

$$I_R(R) = RQkF_R(z)\delta^{-1}; \quad I_S(R) = QF_S(z)\delta^{-1}; \quad z = T_1T_2G^2k^2R^2\hbar^{-2},$$

$$Q = NG^2(4\rho v^2\hbar)^{-1}; \quad F_R(z) = [1+\gamma(1+z)^{1/2}]\{\epsilon^2+[1+\gamma(1+z)^{1/2}]^2\}^{-1}\gamma S_z^\circ \tag{3}$$

$$F_S(z) = \epsilon\{\epsilon^2+[1+\gamma(1+z)^{1/2}]^2\}^{-1}S_z^\circ; \quad \gamma = (\delta T_2)^{-1}; \quad \gamma << 1,$$

$$S_z^\circ = -\frac{1}{2}\,\mathrm{th}(\hbar\omega_0 T^{-1}),$$

where δ is the line width, N the concentration of the paramagnetic centers, G the spin-phonon coupling, v the sound velocity, ρ the density, T_1 and T_2 the times of longitudinal and transverse relaxation, $\varepsilon = (\omega_0 - \omega)\delta^{-1}$, ω_0 the resonant frequency, and ω the hypersound frequency.

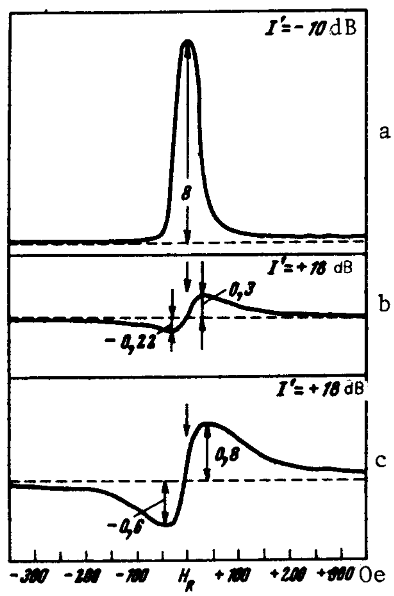
Following the method developed in [1], we obtain a solution of (1) and (2) in the form of a weakly-divergent beam with variable radius of curvature. The self-trapping condition, under which the diffraction divergence is offset by the compression of the beam as the result of the change in the distribution of the phase velocity of the wave over the cross section, takes the form

$$2(ka)^{-2} + \gamma/\Delta v)v^{-1}(z')^{1/2}\epsilon = 0; \quad (\Delta v)v^{-1} = I_S(z'); \quad z' >> 1$$

$$\gamma(z')^{1/2} < 1,$$

where $(\Delta v)v^{-1}$ is the relative change of the phase velocity of the hypersound, $z' = z|_{R = R_0}$, and R_0 is the amplitude of the beam at the entrance (x = 0). It is seen from (4) that self-trapping of the wave is possible only on one side resonance, when $H < H_R$ ($\varepsilon < 0$). Since $(\Delta v)v^{-1}$ and $z' \sim G^2$, self-trapping is favored by the presence of magnetic centers having a large electron-phonon coupling.

3. The self-trapping effect is observed when an intense hypersonic beam of frequency 9.4 GHz propagates in a corundum crystal doped with Ni^{3+} ions, which have a strong electron-phonon coupling due to the orbital degeneracy of the ground state (^2E). A large coupling ($G > 10^3$ cm^{-1}) is obtained for a longitudinal hypersonic wave along the twofold symmetry axis. A thin hypersonic beam of round cross section, with diameter 0.5 mm at the entrance to the crystal, was propagated in the crystal in this direction. Methods of pulsed excitation of hypersound and of registration of the resonance absorption in a magnetic field were described earlier [2]. The intensity of the hypersound in the beam could be varied in the range 5×10^{-5} – 10 W/cm^2. When the intensity of the hypersound is low, linear resonance absorption is observed. With increasing intensity I, $\alpha(H_R)$ decreases monotonically and reaches its half-value at $I = I_S$. Further increase of I leads to a deformation of the resonance curve. The curve becomes maximally deformed at $H < H_R$, and the maximum of $\alpha(H)$ shifts towards stronger magnetic fields. When $I > I_n$, the resonant absorption at $H < H_R$ becomes negative (see the figure), whereas at $H > H_R$ the value of α increases. The negative absorption at fixed I increases with the number of the echo signal.



Resonant absorption curves of hypersound in corundum doped with Ni^{3+}. The extremal values of α in decibels are indicated at the arrows. The intensity of the hypersound in the upper right corner is given (in decibels) relative to $I = I_S$ = 10 mW/cm^2; a, b) first echo signal (accoustic path length L = 3 cm), c) third echo signal (L = 9 cm). T = 4.2°K. H_R = 3.4 kOe. The nonresonant absorption of the hypersound is α_0 = 3 dB/cm.

These data can be readily explained on the basis of the effect of self-trapping of the hypersonic beam in the resonance region. When $I < I_s$, the phase velocity does not vary over the cross section of the beam, and the resonant absorption is observed against the background of a general constant nonresonant absorption, part of which is made up of the hypersound loss due to the diffraction divergence. Further increase of I causes the beam, which has initially at the entrance to the crystal a transverse distribution of the elastic-oscillation amplitude, to become inhomogeneously saturated. As a result, an amplitude-dependent distribution of the phase velocity is produced in this direction. When $H < H_R$ ($\varepsilon < 0$), this propagation is such that the beam contracts and the diffraction divergence is decreased thereby. On going through the resonance, the sign of the dispersion of the phase velocity is reversed. Therefore for $H > H_R$ ($\varepsilon > 0$) the distribution of the phase velocity increases the divergence and there is no self-trapping. The beam contraction gives way to expansion because of the deformation of the resonance curve and the shift of the maximum towards $H > H_R$. When $I = I_{cr}$, the contraction of the beam at $H < H_R$ and $|\varepsilon| = 1$ becomes equal to the diffraction divergences, corresponding to the decrease in the nonresonant absorption by an amount equal to the diffraction loss. This decrease is registered in the form of a negative absorption, equal in modulus to the value of these losses. An estimate of I_{cr}, corresponding to z'_{cr} determined from (4), agrees with the measured value of I at negative absorption (see the figure). The value of the negative resonant absorption agrees also with the diffraction losses obtained on the basis of a calculation for the employed beam geometry and the length of the acoustic path and the crystal [3].

It should be noted that when $I \gg I_{cr}$ hypersound can become self-focused in a medium with resonant centers, and can also become unstable against perturbation in the propagation constants over the cross section of the beam, so that the beam can become stratified.

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