CONTRIBUTION TO THE NONLINEAR THEORY OF INDUCED SCATTERING OF A MONOCHROMATIC WAVE IN A PLASMA

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The theory of induced scattering of wave packets having a wide frequency spread and a random phase has by now been fairly well developed. Its results, however, cannot be used to explain many laboratory and ionosphere experiments in which a monochromatic wave interacts initially with a plasma. The wave amplitudes are frequently so large that the hydrodynamic stage of induced scattering (also called modified decay instability) develops, wherein $\gamma_H > \kappa v_{T\alpha}$, where γ_H is the increment of the induced scattering, $\vec{\kappa} = \vec{k}_1 - \vec{k}_2$, $\vec{k}_1$ and $\vec{k}_2$ are the wave vectors of the incident and scattered waves, respectively, and $v_{T\alpha}$ is the thermal velocity of the scattered particles. We present here certain results of an investigation of the nonlinear stage of hydrodynamic scattering of a monochromatic wave.

As already noted in [1], the analogy with two-stream instability of oscillations in a plasma is useful in this case. The induced scattering is due to the interaction of a moderate-strength incident wave that is quadratic in amplitude and a moderate-strength scattered wave (beats at the difference frequency $\Omega = \omega_1 - \omega_2$), on the one hand, and electrons or ions, on the other. During the kinetic stage, the scattering is due to a small group of resonant particles, whereas in the hydrodynamic stage all the plasma particles interact with the beats. The instability due to induced scattering can be saturated by two effects: phase mixing of the particles trapped in the potential well of the electric field of the beats, and changes in the phase velocity of the beats, which cause the particles to shift from the decelerating to the accelerating phase of the field. Which of the indicated causes prevails depends on the ratio of the maximum oscillation frequency of the particles in the potential well of the beats, $\Omega^B_{max} = (\kappa^2\Phi_{max}/m)^{1/2}$ (see (1)) to the instability increment γ_H. If $\Omega^B_{max} > \gamma_H$, the limitation on the beat amplitude is imposed by the phase mixing of the captured particles, and the maximum amplitudes of the scattered waves can be estimated, just as in the case of two-stream instability, from the relation $\Omega^B \simeq \gamma_H$. Usually this case is realized in scattering by electrons. If $\Omega^B_{max} \ll \gamma_H$, which is typical of scattering by ions, the principal role is played by effects of the change of the phase velocity of the beats; these effects arise when the plasma density is modulated.

To obtain quantitative relations, we consider a two-mode regime in which two quasisinusoidal waves take part in the interaction, one the initial wave and the other corresponds, for example, to the wave having the maximum increment. This regime is crucial for the understanding of the general picture of induced scattering of a monochromatic wave.

The system of equations describing the nonlinear interaction of two quasimonochromatic waves due to induced scattering in an isotropic plasma assumes the simplest form in two limiting cases, in scattering by electrons, when $\kappa r_{de} \gg 1$, and in scattering by ions, when $\kappa r_{de} \ll 1$. Omitting the details of the derivation, we present directly the equations for the complex amplitudes A_1 and A_2 of the interaction waves

$$\frac{dA_1}{dt} = -ra_1A_2\frac{1}{2\pi}\int_{-\pi}^{\pi} d\psi_0 \int_{-\infty}^{\infty} dv_0 f_\alpha(v_0)e^{-i\psi(\psi_0,\, v_0)},$$

$$\frac{dA_2}{dt} = -ia_2A_1\frac{1}{2\pi}\int_{-\pi}^{\pi} d\psi_0 \int_{-\infty}^{\infty} dv_0 f_\alpha(v_0)e^{i\psi(\psi_0,\, v_0)}, \qquad (1)$$

$$\ddot{\psi} = \mathrm{Re}\left(i\frac{\kappa^2\Phi_e}{m_\alpha}e^{i\psi}\right).$$

Here
$$a_{1,2} = \frac{\omega_{pe}^2}{\omega_{1,2}}\left[\frac{1}{\omega}\frac{\partial}{\partial\omega}(\omega^2\epsilon)\right]^{-1}_{1,2}, \quad \psi = \Omega t - \kappa z, \quad \Phi_e = \frac{e^2A_1A_2^*}{2m\omega_1\omega_2},$$

ω_{pe} is the electron Langmuir frequency, the index α determines the type of particle from which the scattering takes place, and $f\alpha(\vec{v}_0)$ is the initial velocity distribution function of these particles.

The first two equations of the system (1) yield, in particular, a conservation law for the number of the quanta of the interacting particles. It is also easy to obtain from (1), in the approximation linear in the amplitude A_2, expressions for the linear increments [1].

The system (2) assumes a form that is easier to analyze and interpret by introducing real wave amplitudes and phases $\tilde{A}_{1,2} = \tilde{A}_{1,2}\exp(i\phi_{1,2})$ and changing over to the following dimensionless variables

$$\Phi = \frac{\tilde{A}_1\tilde{A}_2}{N_{10}}(a_1a_2)^{1/2}, \quad \theta = \frac{N_1 - N_2}{N_{10}}, \quad \phi = \phi_1 - \phi_2, \quad \tau = \gamma_H t, \qquad (2)$$

where the number of quanta is given by the relation

$$N_{1,2} = \frac{\tilde{A}^2_{1,2}}{\omega_{1,2}}\left[\frac{\partial(\omega^2\epsilon)}{\omega\,\partial\omega}\right]_{1,2},$$

N_{10} is the initial number of quanta, and

$$\gamma_{H\alpha} = \omega_1\left(\frac{\kappa^2 e^2 A^2_{10} a_2}{2m_e^2\omega_1^2\,\omega_2}\;\frac{m_e}{m_\alpha}\right)^{1/3}.$$

As a result we get

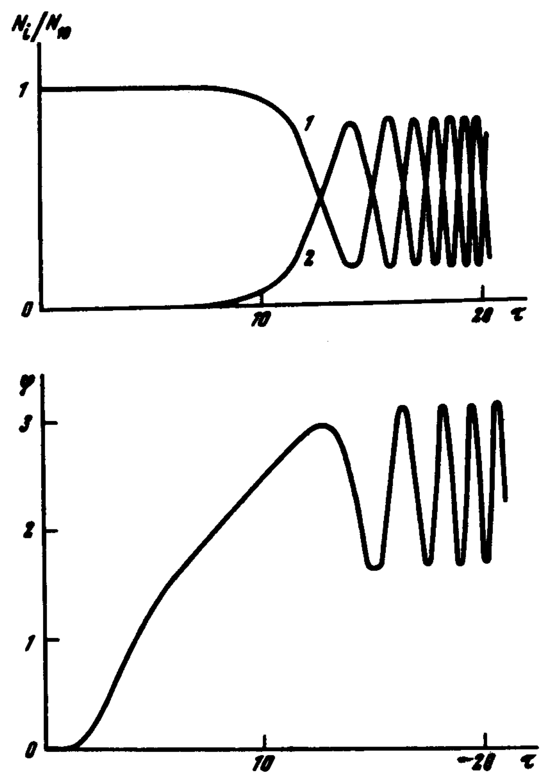
$$\dot{\Phi} = \delta_\alpha \theta I_s\,, \qquad \dot{\theta} = -4\delta_\alpha \Phi I_s\,,$$

$$\Phi\dot{\phi} = \delta_\alpha \theta I_c\,, \qquad \ddot{\psi} = -\delta_\alpha^{-1}\sin(\psi + \phi)\,,$$

$$I_c = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\psi_0 \int dv_0\, f_\alpha(v_0)\cos[\psi(\psi_0, v_0) + \phi]\,, \qquad (3)$$

$$I_s = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\psi_0 \int dv_0\, f_\alpha(v_0)\sin[\psi(\psi_0\, v_0) + \phi]\,.$$

The system (3) has a universal form and depends in the hydrodynamic limit only on one parameter $\delta_\alpha = \omega_{pe}/4\gamma_H\sqrt{\omega_1\omega_2}$. In induced scattering of an electromagnetic wave by electrons, which takes place in a rarefied plasma $\kappa r_{de} \gg 1$, we have $\delta_e \ll 1$ when $\gamma_H \geq \omega_{pe}$. In this case the saturation sets in before the amplitude of the initial wave has time to change, and consequently the population difference θ can be regarded as constant. The system (3) then coincides exactly with the equations describing the two-stream instability in the single-mode regime [2, 3]. The amplitude of the growing wave, obtained with the aid of numerical integration of (3), coincides in order of magnitude with the estimate given above from the frequency of the oscillations of the trapped particles, and is evidence of early saturation of the two-mode regime. This should be followed by a satellite instability that leads to a rapid expansion of the wave spectrum, to heating of the electrons, and to a transition of the instability to the kinetic stage.



The situation is different in the case of scattering by ions in a sufficiently dense plasma, when $\delta_i \gg 1$. A typical time dependence of the amplitudes of the interacting waves, obtained neglecting the thermal scatter of the ions, is shown in the figure. In the calculating we used the parameter $\delta = 50$ and chose the following initial conditions: $\tau = 0$, $\theta_0 = 1$, $\phi_0 = 0.1 f(v_0) = \delta(v_0)$. It is seen from the figure that almost the total energy of the initial wave is transferred to the scattered wave, and the subsequent process is quasiperiodic. Allowance for the thermal motion of the ions during the hydrodynamic stage does not change the character of the interaction and proves that the nonlinear effects of the change of the phase velocity of the beats are connected with bunching of the untrapped particles that interact adiabatically with the field.

The satellite instability should apparently play an important role also in the case of scattering by ions. However, unlike the plasma-beam interaction, this instability can lead under definite conditions not to turbulization but to generation of phased spatial harmonics of the initial wave.

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