EMISSION OF HARD PHOTONS IN e^+e^- ANNIHILATION INTO HADRONS AT HIGH ENERGIES

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It is shown that at high energies the inclusive process of e^+e^- annihilation into hadrons, accompanied by emission of a hard photon from the lepton, is significant at limited transverse dimensions of the singled-out hadron and dominates over the corresponding process without photon emission.

We show in the present paper that at high energies the e^+e^- annihilation with a singled-out hadron h, accompanied by emission of a hard photon q' by the lepton k(k'), $e^-(k') + e^+(k') \to h(p) + \gamma(q') +$ hadrons, is significant for a hadron with a limited transverse momentum, and is even predominant in comparison with the corresponding phononless process and must be taken into account in the experimental study of the inclusive one-photon e^+e^- annihilation in the colliding e^+e^- beams of the near future.

The investigated process is described by the two diagrams of the figure. The differential cross section of this process contains structure functions $\bar{W}_{1,2}(q^2, \nu)$ ($\nu = pq$), describing the vertex of the transition of a virtual photon q into a singled-out hadron with 4-momentum $p(P_0, p_\perp, p_\parallel)$ and the remaining hadrons X.

Since we do not know the behavior of the functions $\bar{W}_{1,2}(q^2, \nu)$ at large q^2, the integration with respect to the direction of the virtual photon is possible only with logarithmic accuracy. A feature of the diagrams of the figure is the presence of t- and u-channel poles with respect to the electron. Using the fact that at

$$q^2 > q^2_{1,2\,min} = \frac{s(p_0 \mp p_\parallel)}{\sqrt{s} - p_0 \mp p_\parallel}$$

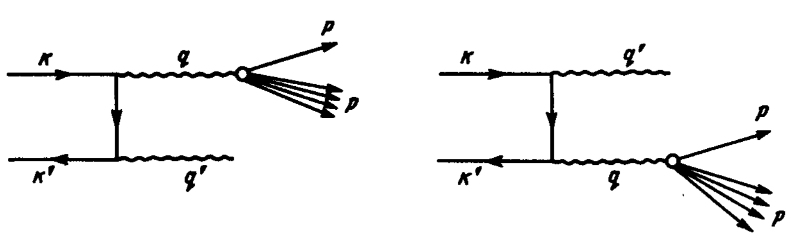
these poles lie near the physical region when integrating with respect to angle between q and p, we obtain after integrating with respect to this angle, with logarithmic accuracy:

$$p_0 \frac{d\sigma}{dp} = \frac{\alpha}{2\pi s} \frac{1 \quad q^4/s^2}{1 - q^2/s} \ln\left(\frac{s}{m_2^2}\right) dq^2 \left\{ \frac{\alpha^2}{q^4}\left[\bar{W}_1(q^2, \nu_1) + \frac{p_\perp^2}{2\mu^2}\bar{W}_2(q^2, \nu_1)\right] + (p_\parallel \to -p_\parallel)\right\}, \qquad (1)$$

where we have separated in the curly brackets, in accord with [1], the inclusive cross section for the process $e^+e^- \to h + X$. Here μ is the hadron mass,

$$2\nu_1\sqrt{s} = (s + q^2)p_0 - (s - q^2)p_\parallel .$$

We are interested only in the contribution from the hard photons ($q'_0 > q'_{0min} = (s - q^2_{max})/2\sqrt{s}$). It is easily seen that in the integration with respect to q^2 from $q^2_{1,2min}$ to $q^2_{max} \simeq s$, the term resulting from q^2/s in (1) are small with power-law accuracy ($\sim 1/s$), if we assume that $\bar{W}_{1,2}(q^2, \nu)$ is bounded at large q^2, so that the main contribution to the integral with respect to q^2 is made by small q^2, and the order of magnitude of the contribution made to $p_0(d\sigma/dp)$ by the hard-photon emission will be $(\alpha^3/sq^2_{1,2min}\ln(s/m_e^2)$. Thus one should expect (with allowance for the smoothness of the structure functions) the ratio r of the inclusive cross section with the emission of a hard quantum to the corresponding cross section without emission to be of the order of



$$\frac{\alpha s}{q^2_{1,2\,min}} \ln\left(\frac{s}{m_2^2}\right).$$

We present a general formula, obtained by integrating the differential cross section in the logarithmic approximation. It is convenient here to introduce in place of q^2 the variable $\omega_1 = 2\nu_1/q^2$ and the functions $\bar{F}_1(\omega_1, \nu_1) = \bar{W}_1(q^2, \nu_1)$ and $\bar{F}_2(\omega_1, \nu_1) = (\nu_1/\mu_2)\bar{W}_2(q^2, \nu_1)$. Then

$$p_0 \frac{d\sigma}{dp} = \frac{\alpha^2}{s^2} \frac{\alpha\sqrt{s}(p_0 + p_\parallel)}{2\pi(p_\perp^2 + \mu^2)} \ln\left(\frac{s}{m_e^2}\right) \left\{\left[\int_{\omega_0}^{1} \bar{F}_1(\omega, \nu_1)\, d\omega + \frac{(p_0 + p_\parallel)p_\perp^2}{\sqrt{s}(p_\perp^2 + \mu^2)} \int_{\omega_0}^{1} \bar{F}_2(\omega, \nu_1)\left(1 - \frac{p_0 + p_\parallel}{\omega\sqrt{s}}\right) d\omega\right] + [p_\parallel \to -p_\parallel]\right\}. \quad (2)$$

Formula (2) is valid for bounded $p_\perp$:

$$\frac{\mu^2}{p_\perp^2 + \mu^2} \ln\left(\frac{s}{m_e^2}\right) >> 1.$$

In order for the effect to be appreciable, p_0 should not be too close to the end point of the spectrum ($2p_0/\sqrt{s} \equiv \omega_0 < 1$).

We consider several particular cases.

1. $p_0 << \sqrt{s}$. In this case $q^1_{1,2\min} >> \mu^2$ both for a nonrelativistic hadron ($q^2_{1,2\min} = p_0\sqrt{s}$, $p_0 \simeq \mu$) and for a relativistic one ($q^2_{1,2\min} \simeq (\sqrt{s}/2p_0)(p_\perp^2 + \mu^2)$, $p_0 >> \mu$). We can assume scaling for $F_{1,2}$, and we have for r

$$r = \frac{\alpha}{\pi} \frac{\sqrt{s}\, p_0}{p_\perp^2 + \mu^2} \frac{\int_0^1 \bar{F}_1(\omega)\, d\omega}{\bar{F}_1(2p_0/\sqrt{s})} \ln\frac{s}{m_e^2} \sim \frac{\alpha\sqrt{s}\, p_0}{\pi(p_\perp^2 + \mu^2)} \ln\left(\frac{s}{m_e^2}\right).$$

For an e^+e^- collision with $E_0 = 5$ GeV ($s = 4E_0^2$) we have $r \sim 0.5[p_0/(p_\perp^2 + \mu^2)]$. For $E_0 = 20$ GeV we get $r \sim 2p_0/(p_\perp^2 + \mu^2)$. (The momenta and mass are here in units of GeV).

2. Let the hadron carry away an appreciable fraction of the lepton energy ($p_0 = \omega_0\sqrt{s}/2$),

$$|x| = \frac{2|p_\parallel|}{\sqrt{s}} \approx \omega_0 < 1.$$

Here $q_{1,2\min} \simeq (p_\perp^2 + \mu^2)/[|x|(1 = |x|)]$. We cannot expect scaling, but assuming $\bar{F}_i(\omega, \nu)$ to be small, r is of the order of $[\alpha s/(p_\perp^2 + \mu^2)]\ln(s/m_e^2)$. We see that in spite of the uncertainties of $\bar{F}_i(\omega, \nu)$, we can state in the case of finite $|x| < 1$ and bounded $p_\perp$ that the process considered here predominates appreciably over the process $e^+e^- \to h + X$ at high energies.

Let us trace the distribution of $p_0(d\sigma/dp)$ with respect to x ($1 > x \geq 0$). Near $x = 0$, the main contribution is made by hadrons with $p_0 << \sqrt{s}$. With increasing x, the inclusive cross section increases and reaches at finite $x < 1$ a maximum, beyond which it drops to zero at $x = 1$. The positions of these maxima cannot be determined without knowing $\bar{F}_i$, but it can be seen that they are practically independent of s. The ratio of the heights of these maxima to the height of the distribution at $x = 0$ is of the order of $\sqrt{s}/(\sqrt{p_\perp^2 + \mu^2})$, i.e., it increases with energy at fixed $p_\perp$.

It would undoubtedly be of interest to observe this picture, which was qualitatively considered in [2] for an e^+e^- annihilation process of higher order in α into two hadron beams via virtual photons emitted by the leptons, and which is qualitatively confirmed in the present article.

We note in conclusion that, unlike the inclusive cross section considered here, the cross section of the process $e^+e^- \to \gamma + X$ without separation of a hadron, which is equal to

$$\frac{d\sigma}{dq^2} = \frac{\alpha}{\pi s} \ln\left(\frac{s}{m_e^2}\right) \sigma_{e^+e^- \to x}(q^2),$$

offers no particular gain at large s, provided that the cross section of $e^+e^- \to x$ does not decrease rapidly with s, which is not very likely and contradicts the existing experiments.

[1] V. N. Baier, V. Fadin, and V. A. Khoze, Proc. of Eigth Summer School, Leningrad Institute of Nuclear Physics, 1972.
[2] H. Cheng and T. T. Wu, Phys. Lett. 41B, 375 (1972).