USE OF THE VAVILOV-CERENKOV EFFECT TO SEPARATE A DIFFERENCE FREQUENCY

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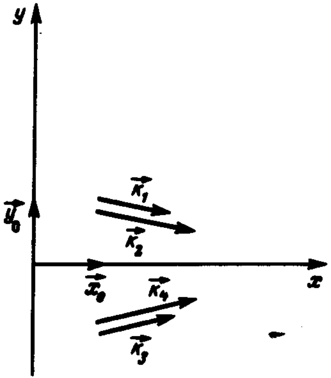
Submitted 2 July 1973
ZhETF Pis. Red. 18, No. 3, 211 — 213 (5 August 1973)

We consider here a new scheme of separating a difference frequency, based on the use of the Cerenkov radiation from bound charges induced in a material medium by an alternating magnetic field.



Let us consider the geometry (Figure) of a field produced in a material medium by superposition of four plane monochromatic electromagnetic amplitude waves whose wave vectors make a small angle α with the ox axis, and the electric field intensity is in the plane of the figure:

$$\mathbf{E} = E_0 \Big\{ (\mathbf{x}_0 \sin\alpha + \mathbf{y}\cos\alpha)\,\mathrm{Re}\Big[e^{i(\mathbf{k}_1 \mathbf{r} - \omega_1 t)} + e^{i(\mathbf{k}_2 \mathbf{r} - \omega_2 t)} \Big] + (-\mathbf{x}_0 \sin\alpha + \mathbf{y}_0 \cos\alpha) \times \mathrm{Re}\Big[e^{i(\mathbf{k}_3 \mathbf{r} - \omega_1 t)} + e^{i(\mathbf{k}_4 \mathbf{r} - \omega_2 t)} \Big] \Big\},$$

where

$$\mathbf{k}_1 = k_1(\mathbf{x}_0 \cos\alpha - \mathbf{y}_0 \sin\alpha),$$
$$\mathbf{k}_2 = k_2(\mathbf{x}_0 \cos\alpha - \mathbf{y}_0 \sin\alpha),$$
$$\mathbf{k}_3 = k_1(\mathbf{x}_0 \cos\alpha + \mathbf{y}_0 \sin\alpha),$$
$$\mathbf{k}_4 = k_2(\mathbf{x}_0 \cos\alpha + \mathbf{y}_0 \sin\alpha).$$

Neglecting terms of second order in α, we obtain after some transformations

$$\mathbf{E} = 4E_0 \Big[\mathbf{x}_0\, \alpha \sin(k_0 x - \Omega t) \cos\frac{kx - \omega t}{2} \sin(\alpha k_0 y) + \mathbf{y}_0 \cos(k_0 x - \Omega t) \cos\frac{kx - \omega t}{2} \cos(\alpha k_0 y) \Big], \tag{1}$$

where

$$k_0 = \frac{1}{2}(k_1 + k_2), \quad \Omega = \frac{1}{2}(\omega_1 + \omega_2), \quad k = k_2 - k_1, \quad \omega = \omega_2 - \omega_1.$$

Expression (1) describes the field in the region in which the slowly varying quantity $\sin(k\alpha y/2)$ is much less than 1.

In a nonlinear quadratic medium, this field leads to the appearance of a system of bound charges with density $\rho = -\mathrm{div}\,\vec{P} = -\mathrm{div}(\chi \vec{E}^2)$. The system of charges will obviously have a periodic structure with unit-cell dimensions $X_0 = 2\pi/k$ and $Y_0 = \pi/\alpha k_0$, and will move with a velocity $\omega/k = c/n$, where n is the refractive index of the medium in the frequency range from ω_1 to ω_2. From this system we can obtain Cerenkov radiation at a frequency ω and of its multiples, under the condition that the propagation velocity of the electromagnetic oscillations at these frequeicies is less than c/n [1]. This condition is easy to satisfy. In the case of ionic crystals, for example, it suffices to have the frequencies Ω and ω lie on opposite sides of the infrared absorption region.

When calculating the power of the Cerenkov radiation, it must be taken into account that, unlike in the classical case, the real velocity of the moving charges may not have the same direction as the velocity of the moving wave of the bound charges. In particular, if the properties of the medium are such that the ratio of the polarization-vector components is

$$\frac{P_x}{P_y} \approx \frac{E_x^2}{E_y^2} \approx \alpha.$$

then, consequently, the bound-charge current is directed along the oy axis, although the charge wave travels along the ox axis. A calculation of the Cerenkov effect with allowance for this difference shows that the front of the wave radiated by a unit cell of the system is made up of two half-planes that intersect at double the Cerenkov angle θ ($\sin\theta = n/n_\omega$) along the unit-cell axis. In order for the power radiated at the frequency ω to be maximal, one must put

$$\mathrm{tg}\,\theta = \frac{Y_0}{X_0} = \frac{n}{n_\omega \sqrt{1 - \left(\frac{n}{n_\omega}\right)^2}}, \tag{2}$$

whence

$$\alpha = \frac{k n_\omega \sqrt{1 - \left(\frac{n}{n_\omega}\right)^2}}{2 k_0 n}.$$

The maximum power radiated by a multilayer system of charges is

$$W = \frac{(4\omega H \chi E^2)^2 L}{\pi c \sqrt{n_\omega^2 - n^2}} = \frac{(\omega \chi W_0)^2 L}{n^2 c \sqrt{n_\omega^2 - n^2}},$$

where H is the height of the multilayer system and is bounded by the condition $\sin(k\alpha H/2) \ll 1$, L is its length, and W_0 is the total power radiated at the frequencies ω_1 and ω_2. A numerical estimate shows that the radiated power can reach several hundred watts at $\chi = 10^{-6}$ cgs esu, W = 10 MW, and ω in the 10^{13} Hz range.

In the general case, the calculations become somewhat more complicated, since it is necessary to allow for the tensor dependence of $\vec{P}$ on $\vec{E}$.

The conversion efficiency can be increased by placing the nonresonant medium in a cavity tuned to the frequency ω and operating at a mode corresponding to the structure of the field (1).

It should be noted that satisfaction of condition (2), and consequently a guarantee of in-phase interaction of the generated radiation with the polarization of the nonlinear medium, is possible in any medium, including one in which it is impossible to obtain coherent wave interaction in the usual sense.

The authors thank B. M. Bolotovskii for interest in the work and for a discussion.

[1] G. A. Askar'yan, Zh. Eksp. Toer. Fiz. 42, 1360 (1960); 45, 643 (1963) [Sov. Phys.-JETP 15, 943 (1960); 18, 441 (1964)].

ERRATUM

In the article by D. V. Vlasov and I. L. Fabelinskii, Vol. 17, No. 9, p. 343, line 3 of the text from the bottom, reads "...$g_{yz}^{(\omega)}$" in place of "...$g_{yx}^{(\omega)}$."