Lifetime of pulsating solitons in certain classical models

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The behaviors of the spherically-symmetrical quasi-solitons of the Higgs field and of the sine-Gordon equation are qualitatively similar. The greater part of the energy of quasi-planar solitons is emitted within several pulsations. The lifetime of the "single-scale" energy clusters is estimated.

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An important current problem is the search for the nonlinear relativistically-invariant equations stable spatial solutions that could be interpreted as classical models of finite-size particles. A possibility of just this type is discussed in[1,2] for the existence of long-lived pulsating meson solitons, the dynamics of which was described in a spherically symmetrical (ss) geometry by the Higgs field equation

$$u_{tt} - \Delta_{rr} u - m^2 u + g^2 u^3 = 0. \qquad (1)$$

The model of[2] is all the more of interest, since the quasiplanar solitons

$$u = \frac{m}{g} \,\mathrm{th} \frac{m(r - R_0)}{\sqrt{2}}, \qquad R_0 >> l \approx \frac{1}{m}, \qquad (2)$$

which specify the initial state of the bubble, are apparently stable with respect to transverse (angular) perturbations, in contrast to certain other types of relativistic solitons.[3] Unfortunately, it is impossible to estimate analytically for this model the value of the energy radiated at infinity and the associated lifetime of the bubbles. In addition, it remains unclear whether the bubble returns, after reflection from the center, to the initial soliton state (or close to it).[1)] In this paper we investigate the dynamics of bubbles, using a computer solution of (1) by the grid method and a difference scheme that conserves, with high accuracy, the energy integral

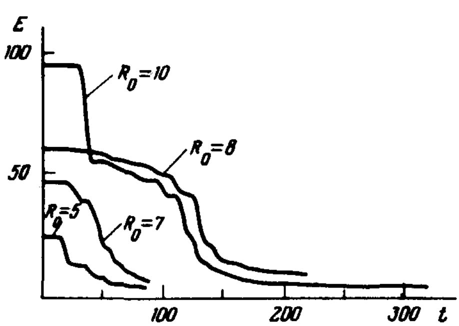

FIG. 1. Plot of $E(t)$ for Eq. (1) at different R_0.

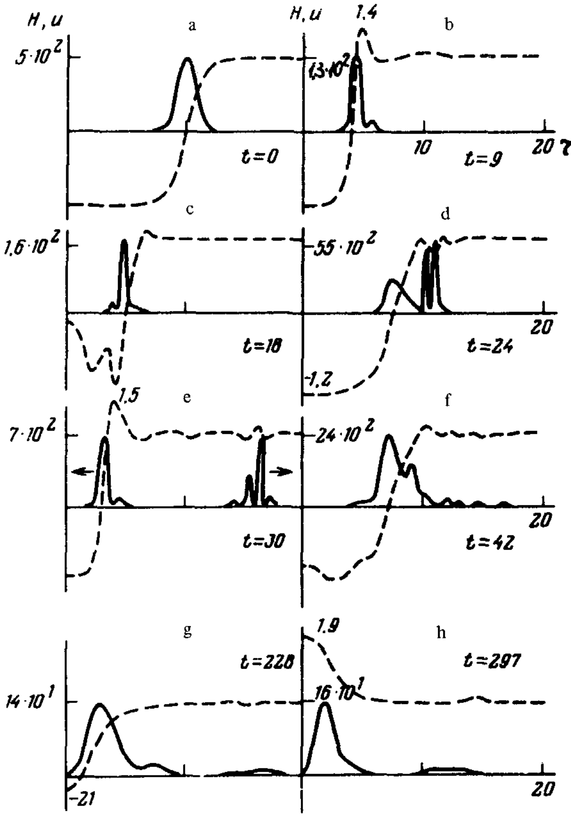

FIG. 2. Typical pattern of bubble evolution ($R_0 = 10$).

$$E = \int_0^{r_m} H\,dr, \qquad H = \frac{1}{2} r^2 \left[(u_t)^2 + (u_r)^2 + \frac{1}{2}(u^2 - 1)^2 \right] \tag{3}$$

upon substitution of the boundary conditions $u(r_m) = m/g$ that "lock" the wave energy in a sphere $r < r_m = 2R_0$. We shall henceforth admit the possible emergence of the radiated waves from this sphere; in the course of the calculation we determine the total flux $Q(t) = -r^2 u_t u_r$ of the energy that is carried away at $r \approx r_m$ and obtain the function $E(t)$ and the distribution $H(r,t)$. The calculation is carried out in terms of transformed variables r and t, with $m = 1$ and $g = 1$.[2)]

The evolution depends substantially on R_0, as is seen from Fig. 1. The most regular reflection picture is observed at $R_0 = 8$: in the first two compression and expansion cycles, the system returns with good accuracy to the initial soliton state (2); the return takes place, albeit with less accuracy, in the next three pulsations. Starting with the third cycle, however, disengagement of ever-increasing batches of energy from the main cluster is observed, and the flux Q out of the sphere $r = r_m$ increases. After the sixth reflection ($t \approx 110$), the cluster splits into several spherical layers: part of them move towards the boundary r_m and remove from the region $r < r_m$ approximately half the energy $E(t)$ concentrated in this region at this time. It appears that a near-maximum ratio $T/R_0 \approx 15$ is obtained at $R_0 = 8$ [T is the lifetime of the discussed pulsating solutions with initial data (2)].

At $R_0 = 5, 7, 10$, and 15, the field energy breaks up into individual spherical layers even after the first reflection, and no return to the soliton state (2) is observed. The energy involved in the second compression-expansion cycle is much lower than the initial value, and $E(t)$ decreases rapidly as a result of powerful radiation, starting with $t_{rad} \sim 3R_0$. Figure 2 shows characteristic instants in the evolution of a bubble, starting with $R_0 = 10$. At $t \approx 200$ we have $E(t) \approx 0.1 \times E(0)$; the value of $Q(t)$ becomes small by that time. A "single-scale" energy cluster is produced near the center (its characteristic half-width L is of the order of the distance R_1 to the center), and is described by a nonstationary solution $u(r,t)$ that oscillates about the vacuum value $u_0 = 1$ [Figs. 2(g) and 2(h)]. The lifetime $T_1 = -E/(dE/dt)$ of this state is of the order of 100 times its radius R_1. Whether such oscillating solutions are of interest as models of mesons and "bags" for quarks[4] is still a moot question.

We next investigated, within the framework of the sine-Gordon equation $u_{tt} - \Delta_{rr} u = -\sin u$, in *ss* geometry, the evolution of initial soliton data in the form $u = 4\tan^{-1}[\exp(r - R_0)]$, with $R_0 \gg 1$, for the purpose of finding spatial analogs of the special properties (complete integrability and the ensuing elastic soliton interactions that produce no radiation) that this equation possesses in the planar one-dimensional (x,t) case.[5] In particular, any hindrance on the radiation in the *ss* model could lead to an infinite duration of strictly periodic pulsations. However, calculations carried out at $R_0 = 12$ have revealed the presence of high-power radiation even after the first reflection from the center (the picture of the evolution of the energy clusters agrees qualitatively with that shown in Fig. 1); at $t = 88$, the energy of the system decreased to less than one-half the initial value. No fundamental differences are observed between the evolutions of the bubbles in the models of Eq. (1) and of the sine-Gordon equation. Thus, no unusual properties of the sine-Gordon equation were observed at all in the performed experiments.

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1) The numerical-solution procedure used in[2] has led to unphysical energy absorption and could not answer these questions.
2) We note that the variables r' and t' [2] are connected with r and t by the relations $r = 2r'$ and $t = 2t'$.

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