Effect of acoustic turbulence on the collapse of Langmuir waves

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(Submitted May 12, 1976)
Pis'ma Zh. Eksp. Teor. Fiz. **24**, No. 1, 25–29 (5 July 1976)

It is shown that acoustic pulsations emitted from collapsing cavitons with plasmons, produce an additional mechanism of short-wave conversion-induced transfer of Langmuir waves, and can stabilize the collapse.

PACS numbers: 52.35.Js, 52.25.Gj

An analysis of numerical experiments[1,2] has made it possible to call attention to the following singularities of the collapse of Langmuir waves. When a caviton with plasmons trapped in it reaches sufficiently small dimensions, such that the absorption of the plasma waves by the particle becomes significant, the equilibrium between the high-frequency and gas-kinetic pressures is violated. The excess density is then released in the form of sound waves radiated from the cavitons. A plasma with advanced Langmuir turbulence is not isothermal. In such a plasma, we are faced with the problem of accumulation of energy in the short-wave acoustic pulsations—the pumping to the sound waves takes place with a characteristic modulation-instability growth rate, and the absorption of these pulsations by the resonant electrons is much slower.

The equation for the energy of the acoustic pulsations

$$W_s = n_0 T \sum_k \frac{|\delta n_k^s|^2}{n_0^2}$$

then takes the form

$$\frac{dW_s}{dt} = 2\gamma_M W_L^{\prime} k^2 \lambda_D^2 - 2\gamma_e W_s . \qquad (1)$$

In this equation

$$\gamma_M = \omega_p \sqrt{\frac{m}{M}} \frac{|E_L|^2}{8\pi n_0 T}$$

is the growth rate of the modulation instability,

$$W_L^{\prime} = \frac{|E_L|^2}{8\pi} \left(\frac{k_0}{k}\right)^{3/2}$$

is the energy in the short-wave part of the spectrum of the Langmuir waves (in the three-dimensional case),[3] k is the characteristic wave number in this part of the spectrum,

$$k_0 = \frac{1}{\lambda_D} \sqrt{\frac{|E_L|^2}{8\pi n_0 T}}$$

is the wave number at which energy is pumped into the Langmuir waves, and $\gamma_e = \omega_p (m/M) k \lambda_D$ is the decrement of the sound-wave absorption by the electrons.

In the stationary state, the amplitude of the acoustic pulsations, determined from Eq. (1), is quite large:

$$\sum_k \frac{|\delta n_k^s|^2}{n_o^2} \approx 8\pi\sqrt{\frac{M}{mk\lambda_D}}\left(\frac{|E_L|^2}{8\pi n_o T}\right)^{9/4} . \tag{2}$$

The presence of an intense short-wave sound leads to stabilization of the collapse. This effect is connected with the onset of an additional mechanism whereby plasmons are transferred to the short-wave region (the absorption region) as a result of direct conversion on the acoustic pulsations.

Assuming that the acoustic turbulence is isotropic and weak ($\sum_k |\delta n_k^s|^2/n_0^2 \ll k^4\lambda_D^4$), we obtain the following system of equations for the description of the collapse in the presence of acoustic pulsations:

$$\mathrm{div}\left(i\frac{\partial \mathbf{E}_L}{\partial t} + \frac{3}{2}\omega_p\lambda_D^2\,\nabla\,\mathrm{div}\,\mathbf{E}_L - \frac{\omega_p}{2n_o}\delta n\,\mathbf{E}_L\right) = -\frac{\omega_p}{12}\,\mathrm{div}\left(\sum_k \frac{|\delta n_k^s|^2}{n_o^2 k^2\lambda_D^2}\left(1+\frac{2}{3}i\frac{\Gamma_k}{\omega_p k^2\lambda_D^2}\right)\mathbf{E}_L\right), \tag{3}$$

$$\frac{\partial^2 \delta n}{\partial t^2} - \frac{T_e}{M}\Delta\delta n = \frac{1}{16\pi M}\Delta|\mathbf{E}_L|^2. \tag{4}$$

$\mathbf{E}_L(t,\mathbf{r})$ is the complex amplitude of the Langmuir-wave field, $\mathbf{E}=\frac{1}{2}\mathbf{E}_L(t,\mathbf{r}) \times\exp(-i\omega_p t)+\mathrm{c.c.}$; $n(t,\mathbf{r})$ is the long-wave variation of the density, Γ_k is the decrement of the Landau damping of the plasma wave. In the right-hand side of (3), the first term describes the plasmon frequency shift due to their scattering by the short-wave density fluctuations.

This shift

$$\delta\omega = -\frac{\omega_p}{12}\sum_k \frac{|\delta n_k^s|^2}{n_o^2 k^2\lambda_D^2}$$

is the same for the entire long-wave part of the spectrum, including the pump, and therefore exerts no influence on the dynamics of the modulation instability. The second term corresponds to damping of long-wave plasmons due to their conversion on sound, with a decrement

$$\gamma_{\mathrm{conv}} = \frac{1}{18}\sum_k \Gamma_k \frac{|\delta n_k^s|^2}{n_o^2 k^4\lambda_D^4} .$$

In the stationary state, the energy flux into the short-wave part of the spectrum should be compensated by the absorption by the particles

$$\gamma_M \frac{|E_L|^2}{8\pi} = \Gamma_k W_L^s .$$

From this we determine the characteristic value of k for the absorption region

$$\Gamma_k = \omega_p \sqrt{\frac{m}{M} k^2\lambda_D^3}\left(\frac{8\pi n_0 T}{|E_L|^2}\right)^{1/4} . \qquad (5)$$

From an analysis of the system of equations (3) and (4) it follows that a sufficiently high sound level, such that the plasma-wave damping decrement due to the conversion exceeds the growth rate of the modulation instability, stabilizes the collapse [the well produced by the long-wave fluctuations of the density is not deep enough to trap the plasmons, $\delta n/n_0 \sim (\gamma_M^2/\gamma_{conv}^2) k_0^2\lambda_D^2$].

Thus, formula (2), which determines the level of the acoustic pulsations from the balance of their influx from the cavitons and their absorption by the electrons, is valid only at not too large Langmuir-wave amplitudes:

$$\frac{|E_L|^2}{8\pi n_0 T} \leqslant \frac{|E_L^*|^2}{8\pi n_0 T} = (8\pi)^{-2/3}\left(\frac{81 m}{M}\right)^{1/3} k^2\lambda_D^2 . \qquad (6)$$

In strong fields, the acoustic turbulence influences strongly the dynamics of the collapse and a regime is established wherein the influx of acoustic pulsations due to the collapse is limited to a level for which $\gamma_{conv} = \gamma_m$. Taking (5) into account, we then have in place of (2) the following formula for the acoustic pulsations:

$$\sum_k \frac{|\delta n_k^s|^2}{n_0^2} = 9 (k\lambda_D)^{5/2}\left(\frac{|E_L|^2}{8\pi n_0 T}\right)^{3/4} . \qquad (7)$$

The condition that the acoustic turbulence is weak corresponds to fields $|E_L|^2/8\pi n_0 T < k^2\lambda_D^2$, and, as can be easily seen, coincides with the condition for the existence of the inertial interval $k_0 < k$.

We note that at the acoustic-turbulence level determined by relation (7), the energy of the short-wave Langmuir oscillations produced by the conversion

$$\sum_k |E_k|^2 \sim |E_L|^2 \sum_k \frac{|\delta n_k|^2}{n_0^2} \frac{1}{k^4\lambda_D^4}$$

is of the same order as the energy in the short-wave part of the spectrum of the Langmuir-waves in the subthreshold regime $E_L < E_L^*$, when the short-wave energy transfer is due to the collapse $\sum_k |E_k|^2 \sim (k_0/k)^{3/2} |E_L|^2$.

However, when the threshold value of the electric field E_L^* is exceeded, the rate at which the collapse develops is greatly decreased: $\gamma = \gamma_M(|E_L^*|^2/|E_L|^2)^{3/2}$, as follows from the balance equation for the sound (1). Above the threshold, therefore, the conversion is the main mechanism whereby the short-wave oscillations are produced.

Let us dwell in conclusion on the question of the influence of intense acoustic turbulence on the rate of energy pumping into the Langmuir wave. The pumping, as usual, is introduced with the aid of the condition $\bar{E} = E_0$ (the bar denotes averaging over the volume of the plasma, and E_0 is the constant amplitude of the pump wave). Then the growth rate of the Langmuir-wave energy is then obtained from the equation

$$\frac{d}{dt}|E_L|^2 = \nu_p E_o^2 , \tag{8}$$

where

$$\nu_p = \frac{\nu_{\text{eff}}}{(\Delta\omega)^2 + \nu_{\text{eff}}^2}\,\omega_p^2\overline{\left(\frac{\delta n}{n_o}\right)^2},$$

$$\nu_{\text{eff}} = \frac{1}{\tau_c} + \frac{1}{9}\sum_k \frac{|\delta n_k^s|^2}{n_o^2 k^4 \lambda_D^4}\Gamma_k, \quad \Delta\omega = \frac{3}{2}\,\omega_p k_o^2 \lambda_D^2 .$$

In these formulas τ_c is the time of decoupling of the phase correlations for the long-wave plasmons, $\tau_c^{-1} \sim (|E_L|^2/8\pi n_0 T)\omega_p$. In our case of supersonic collapse $|E_L|^2/8\pi n_0 T \gg m/M$ the ν_{eff} term due to conversion is small, and we have for the rate of the pump-wave energy dissipation the usual result $\nu_p \sim 1/\tau_c \sim \omega_p k_0^2 \lambda_D^2$.[3]

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