Quantum magnetization jumps in magnets with easy-plane anisotropy

E. V. Rozenfel'd

Institute of Metal Physics, Urals Scientific Center, USSR Academy of Sciences
(Submitted June 3, 1976)
Pis'ma Zh. Eksp. Teor. Fiz. **24**, No. 2, 60–64 (20 July 1976)

It is shown that in paramagnets and in ferromagnets with small T_c, having an anisotropy of the easy-plane type, the dependence of the magnetization on the external magnetic field applied along the anisotropy axis z has at $T=0$ a steplike form (z component) or high peaks (x component).

PACS numbers: 75.30.Gw

The dependence of the magnetization on the external field in the presence of uniaxial anisotropy has been throughly investigated for ferromagnets in which the volume energy is much larger than the anisotropy energy. In particular, if the external field H_0 is applied along the "difficult axis," under the condition

$$H_0 > H_A = \frac{2\mathcal{K}}{\mathcal{M}_s} \tag{1}$$

($\mathcal{K}$ is the anisotropy constant that enters in the expression for the free energy, $\mathcal{M}_s$ is the saturation magnetization, and H_s is the anisotropy field) the magnetization vector is directed along the field. In fields weaker than the anisotropy field, the angle θ between the magnetization vector and the field direction, is determined, as is well known, by the expression

$$\operatorname{tg}\theta = \sqrt{\frac{4\mathcal{K}^2}{H_0^2\mathcal{M}_s^2} - 1}. \tag{2}$$

The derivation of these relations is based in essence on the fact that in weak fields (compared with the exchange field) the total angular momentum of the crystal is a "good" quantum number. Since this number is very large, the angular momentum can be regarded quasi-classically, the effects vanish, and the angle θ in (2) is practically a continuous variable. In the opposite limiting case when there is no exchange interaction, or its energy is low in comparison with the energies of the anisotropy and of the interaction with the external field, only the angular momentum J of an individual magnetic ion is a "good" quantum number. Under these conditions one can expect singularities of a quantum character to appear in the dependence of the magnetization on the field, and the purpose of the present paper is to consider these singularities.

Confining ourselves only to one anisotropy constant, we choose the zero-order Hamiltonian for an individual magnetic ion in the form

$$\hat{\mathcal{H}}_0 = -h_0\hat{J}_z + k\hat{J}_z^2; \quad h_0 = g_J\mu_B H_0, \tag{3}$$

where H_0 is the z component of the external field, $k>0$ is the anisotropy constant, and the remaining symbols are standard. Simple calculation shows the following: 1) The eigenfunctions $|m\rangle$ of $\mathcal{H}_0$ satisfy the equation $\hat{J}_z|m\rangle = m|m\rangle$. 2) With increasing H_0, when the condition

$$\frac{h_0}{2k} = m - \frac{1}{2}\ ; \quad m = J,\ J-1, \ldots;\ m > 0 \tag{4}$$

is satisfied, a change takes place in the ground state $|m-1\rangle \to |m\rangle$. Accordingly, the z component of the magnetization $M_z = g_J\mu_B m$ changes jumpwise at zero temperature (see the figure). When (4) is satisfied, the energies of the states $|m-1\rangle$ and $|m\rangle$ are the same, and the ground state is degenerate.[1)]

If the ground state is degenerate, then at zero temperature an infinitesimally weak perturbation suffices for the appearance of a large magnetization component in a plane perpendicular to the z axis. For paramagnets, we take the perturbation to be the interaction with the (small x component of the external magnetic field H:

$$\hat{V} = -h\hat{J}_x = -g_J\mu_B H\hat{J}_x.$$

For ferromagnets, within the framework of the molecular field method we have $V = -I(\langle J_z\rangle\hat{J}_z + \langle J_x\rangle\hat{J}_x)$, where I is the molecular-field parameter. Assuming that $H \ll h_0 k$ and $I \ll h_0 k$, we obtain the following for ferromagnets at $T=0$, by using perturbation-theory methods with subsequent self-consistency: 3) If the ground state is not degenerate, then the x component of the magnetization is $M_x = g_J\mu_B\langle J_x\rangle \ll g_J\mu_B$ for paramagnets and is equal to zero for ferromagnets. 4) If (4) is satisfied, we have for paramagnets

$$M_x = \tfrac{1}{2} g_J\mu_B\sqrt{(J+m)(J-m+1)};\quad M_z = g_J\mu_B(m-\tfrac{1}{2});\quad \delta\epsilon = h\sqrt{(J+m)(J-m+1)}, \tag{5}$$

and for ferromagnets

$$M_x = g_J\mu_B\sqrt{\frac{1}{4} - \frac{(m-\tfrac{1}{2})^2}{[(J+m)(J-m+1)-1]^2}}\ \sqrt{(J+m)(J-m+1)}\ ;$$

$$M_z = g_J\mu_B\frac{(J+m)(J-m+1)}{(J+m)(J-m+1)-1}(m-\tfrac{1}{2}),\qquad \delta\epsilon = \tfrac{1}{2}I\sqrt{(J+m)(J-m+1)}, \tag{6}$$

where $\delta\epsilon$ is the splitting of the ground state. Thus, in this case the magnetization vector deviates greatly from the direction of the external field. The dependence of M_x on H_0 is shown qualitatively in the figure. (See p. 52.)
5) The ratio of the widths of the M_x peaks to the distance between them is $\sim h/2k$ (or $IJ/2k$). The steps of M_z should be spread out over the same width. 6) The peaks of M_x and the steps of M_z vanish at

$$h_0 > 2k(J-\tfrac{1}{2})\ . \tag{7}$$

Using the known relations (at $T=0$)

$$\mathcal{K} = NkJ(J-\tfrac{1}{2})\ ;\quad \mathcal{M}_s = Ng_J\mu_B J$$

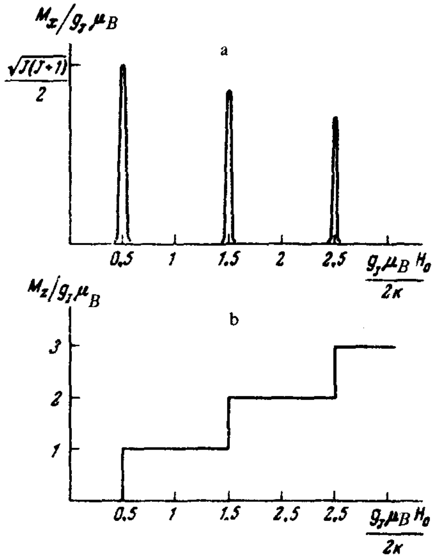

FIG. 1. Dependence of $M_x(a)$ and $M_z(b)$ on the external magnetic field for integer $J(T=0)$. For a half-integer J, the plots should be shifted by 1/2 to the left along the abcissa axis.

(N is the number of magnetic ions per unit volume), it is easy to show the following: 7) Equation (1) is equivalent to (7), and this condition is valid for all I. 8) The ratio M_x/M_z from (5) at $J\gg 1$ and $M\gg 1$ is half the value of (2), because in the ground state of the paramagnet we have $\langle J_x^2\rangle = \langle J_y^2\rangle$.

We note also that phenomena similar to those described here can occur also at $J>3/2$ in easy-axis magnets if the field H_0 is perpendicular to this axis, (these phenomena do not occur at $J=1$ or 3/2, when the problem can be solved exactly).

It is obvious that with increasing I the peaks of M_x (see the figure) broaden and coalesce, and this leads to the existence of an appreciable angle between the field and the magnetization in all fields $H_0<H_A$. A fastened sample is then acted upon by a large torque, while an unfastened sample will rotate through a definite angle. A similar phenomenon occurs when the crystal consists of two magnetic sublattices (e.g., RCo_5), with the exchange weak in one of them (R) and strong in the other (Co). Since the exchange inside the R sublattice is larger than its anisotropy, and the role of the external field is assumed by the exchange with the Co sublattice, the magnetization vector of the Co sublattice is acted upon by an appreciable torque. Some consequences of this fact were considered in rough approximation in [2].

It is also possible that in a definite range of exchange fields a self-consistent calculation can yield a cone of easy directions for the magnetization even in the absence of an external field, in the presence of only one anisotropy constant.

Experimental observation of the dependences of the magnetization on the field, shown in the figure, is a rather complicated task, since it is possible only in strong fields at low temperatures. Sharp peaks of M_x can be observed if the splitting $\delta\epsilon$ of the ground state satisfies three requirements. First, $\delta\epsilon \lesssim k/10$, second, $T<\delta\epsilon$,[2)] and third, $\delta\epsilon$ is much larger than the ground-state splitting due to the Jahn-Teller effect. On the other hand, even to observe two peaks of M_x, and consequently two steps of M_z at integer J, it is necessary to have fields satisfying the condition $h_0 > 3k$. Starting, for example, from the maximum value $H_0 \sim 100$ kOe, we obtain $k \sim 3$ °K and $\delta\epsilon$ should amount to several tenths of a degree. Recognizing that the anisotropy constants of rare-earth metals and in their compounds with metals of the iron group are approximately of this magnitude, and the splitting due to the Jahn-Teller effect in rare-earth metals is $\sim 10^{-2}$ cm^{-1},[3] we arrive at the conclusion that these substances are the most suitable objects for experiment. The required splitting $\delta\epsilon$ at $J \sim 5$ can be obtained with a transverse field $H \sim 1$ kOe. Consequently, the exchange field should be of the same order of magnitude if exchange interaction exists between the rare-earth ions. It is therefore desirable to carry out the measurements on alloys diluted with nonmagnetic La or Y.

The author thanks Yu. P. Irkhin for interest in the work and for very useful discussions.

1)Magnetization jumps in cubic crystals, due to a similar cause, were considered by Cooper.[1]

2)At $T = \delta\epsilon$, the heights of the M_x peaks decrease to approximately one half the values at $T = 0$; the steps of M_x will be clearly distinguishable.

---

1. B. R. Cooper, Phys. Lett. 22, 244 (1966).
2. E. V. Rozenfel'd and Yu. P. Irkhin, Fiz. Tverd. Tela **18**, 367 (1976) [Sov. Phys. Solid State 18, 214 (1976)].
3. S. A. Al'tshuler and B. M. Kozyrev, Electronnyĭ paramagnitnyĭ rezonans (Electron Paramagnetic Resonance), Fizmatgiz, 1961 (Academic, 1961.