Electron liquid in a superstrong magnetic field

L. V. Keldysh and T. A. Onishchenko

P. N. Lebedev Physics Institute, USSR Academy of Sciences
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It is shown that the energy of an electron-hole plasma in ultrastrong magnetic field has, as a function of the density, a minimum in the strong-compression region. The equilibrium density is $n_0 \sim (eHa_{0\,2}/\hbar c)^{8/7} a_0^{-3}$, and the energy per particle is given by $E/n_0 \sim -n_0{}^{1/4} e^2 a_0{}^{-1/4}$, where H is the field intensity and a_0 is the effective Bohr radius.

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We calculate in this paper the energy of high-density electron-plasma in the limit of strong magnetic fields.

We use Coulomb units $e = \hbar = m_0 = 1$ and the unit of the magnetic field intensity is $e^3/\hbar^3 m_0^2 c$.

We assume that the magnetic length is $\lambda = H^{-1/2} \ll 1$ and that the particle concentration satisfies the condition $1/\lambda^2 \ll n \ll 1/\lambda^3$. Then, by virtue of the right-hand side of the inequality, all the electrons and holes are at the lower Landau level, and the transitions between the different Landau levels can be neglected, while the left half of the written inequality is the compressibility condition of the system, inasmuch as the effective volume of the atom (exciton) is $\propto\lambda^2$.[1] In the considered density range, the system is therefore a degenerate Fermi liquid with Fermi momentum $p_0 = 2\pi^2 n\lambda^2 \gg 1$. An analysis of the perturbation-theory series shows that, just as in any compressed system with Coulomb interaction, the principal contribution (in terms of the parameter p_0^{-1}) to the correlation energy is made in this situation by the so-called random-phase-approximation (RPA) diagrams with momentum transfers $|\mathbf{k}| \ll n^{1/3}$. However, as will be shown below, a peculiarity of our problem is the presence of a concentration region in which the main contribution to the energy is made by $|\mathbf{k}| \sim n^{1/4} \gg p_0$, so that the dependence of the correlation energy on the density is qualitatively altered and a minimum of the energy appears.

For simplicity we calculate the energy of the ground state of an electron gas in a magnetic field $\mathbf{H} \parallel OZ$, and write out the final results for an electron-hole plasma with isotropic masses of the electrons and holes. The correlation of the system

$$E_{\rm corr} = \frac{i}{2}\int_0^1 \frac{d\alpha}{\alpha}\int \frac{d^3k\,d\omega}{(2\pi)^4}\left[\frac{\frac{4\pi\alpha}{k^2}\Pi(\alpha,\omega,\mathbf{k})}{1-\frac{4\pi\alpha}{k^2}\Pi(\alpha,\omega,\mathbf{k})} - \frac{4\pi\alpha}{k^2}\Pi_0(\omega,\mathbf{k})\right]$$

is expressed in terms of the exact polarization operator $\Pi(\alpha,\omega,\mathbf{k})$ and its first-order approximation $\Pi_0(\omega,\mathbf{k})$ in the coupling constant α. The contribution of diagrams other than the RPA diagrams is small in the parameter $1/n\lambda^2$, and at momentum transfers on the order of $n^{1/4}\,\Pi(\alpha,\omega,\mathbf{k})$ it reduces to $\Pi_0(\omega,\mathbf{k})\cdot E_{\rm corr}$ is expressed by the well-known formula

$$E_{\rm corr} = \frac{1}{2}\int\frac{d^3k\,d\omega}{(2\pi)^4}\left\{\ln\left[1-\frac{4\pi}{k^2}\Pi_0(i\omega,\mathbf{k})\right] + \frac{4\pi}{k^2}\Pi_0(i\omega,\mathbf{k})\right\},$$

$$\Pi_0(i\omega,\mathbf{k}) = -\frac{1}{4\pi^2\lambda^2}\exp\left(-\frac{k_\perp^2\lambda^2}{2}\right)\frac{1}{k_z}\ln\left[\frac{\omega^2+\left(\frac{1}{2}k_z^2+k_zp_0\right)^2}{\omega^2+\left(\frac{1}{2}k_z^2-k_zp_0\right)^2}\right].$$

Integrating over the transverse momentum with accuracy to $1/n\lambda^2$ and expressing ω in terms of a new independent variable v

$$v = \frac{1}{\pi\lambda^2k_z^3}\ln\frac{\omega^2+\left(\frac{1}{2}k_z^2+k_zp_0\right)^2}{\omega^2+\left(\frac{1}{2}k_z^2-k_zp_0\right)^2},$$

we obtain

$$E_{\rm corr} = -\frac{4p_0^5}{\pi^3\gamma}f(\gamma),\ \text{where}\ \gamma = 1/8\,\pi\lambda^2p_0^3$$

$$f(\gamma) = \int_0^\infty dv\,[(1+v)\ln(1+v)-v]\int_0^{u(v/\gamma)} dx\,x^7\exp\left(-\frac{1}{\gamma}vx^3\right)$$

$$\times\left[1-\exp\left(-\frac{1}{\gamma}vx^3\right)\right]^{-3/2}\left[(1+x)^2\exp\left(-\frac{1}{\gamma}vx^3\right)-(1-x)^2\right]^{-1/2}.$$

$u(v/\gamma)$ is the solution of the equation

$$v/\gamma = (2/u^3)\ln\left|\frac{1+u}{1-u}\right| .$$

At $\gamma \gg 1$ the calculations yield

$$\frac{E_{corr}}{n} = -\frac{2^5\pi^{3/4}}{5[\Gamma(1/4)]^2} n^{1/4} \approx -1.1\, n^{1/4} .$$

The main contribution accumulates in this case in a momentum-transfer region of the order of $n^{1/4}$, and the contribution of momenta smaller than p_0 is of the order of $\max\{n^4\lambda^{10}; n\lambda^2\}\ln\gamma$. At $\gamma \ll 1$ the main contribution to E_{corr} is due to momenta smaller than p_0, and

$$\frac{E_{corr}}{n} = -\frac{1}{64\pi^6 n^2\lambda^6}\ln\frac{1}{\gamma} ,$$

which agrees with the results of [2] apart from the factor $(\ln 2\pi^3 n\lambda^2)(\ln 8\pi^7 n^3\lambda^8)^{-1}$.

The exchange part of the energy is of the order of $-n\lambda^2\ln(1/n\lambda^3)$, and is negligible in the considered density interval. The energy of the ground state

$$\frac{E}{n} = \frac{2}{3}\pi^4 n^2\lambda^4 - \frac{2^5\pi^{3/4}}{5\left[\Gamma\left(\frac{1}{4}\right)\right]^2} n^{1/4}$$

has a minimum $(E/n)_{min} \approx -0.42H^{2/7}$ at $n_0 \approx 0.030H^{8/7}$. For an electron-hole plasma with $m_e = m_h = m$ at $\gamma \gg 1$ we have

$$\frac{E_{corr}}{n} = -\frac{2^5\pi^{3/4}}{5\left[\Gamma\left(\frac{1}{4}\right)\right]^2} 2^{5/4}\frac{m^{1/4}}{\epsilon^{5/4}} n^{1/4} \approx -2.7\frac{m^{1/4}}{\epsilon^{5/4}} n^{1/4} ;$$

$$\left(\frac{E}{n}\right)_{min} \approx -1.0H^{2/7} m^{3/7}\epsilon^{-10/7}$$

at $n = n_e = n_h \approx 0.034m^{5/7}H^{8/7}\epsilon^{-5/7}$, where ϵ is the dielectric constant of the medium. If m_e and m_h are essentially different, then E_{corr} is determined mainly by the large mass, in contrast to the binding energy of the exciton.

Just as in the absence of a magnetic field [3], the spectrum has a gap as a result of the logarithmically diverging diagrams near the Fermi surface. [4,5] The magnets of the gap, however, is negligible because of the high density of the plasma. If m_e and m_h are so different that the assumptions of the present paper are satisfied only for light nuclei, and the mass of the heavy particles is much larger than $n^{1/3}$, then the heavy particles form a Wigner crystal.

We note in conclusion that the result has a general character in view of the universality of the asymptotic form of the polarization operator at large momentum transfers.

The RPA diagrams with momentum transfer on the order of $n^{1/4}$ make a contribution of the order $n^{1/4}$ per particle to E_{corr} in any high-density system in which the characteristic momenta of the particles are much smaller than $n^{1/4}$, regardless of the dimensionality of the system, and in the presence of bound states the characteristic momentum is the reciprocal dimension of the exciton.

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