Effect of absorptivity of a medium on the formation of hard transition radiation

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Very hard quanta, for which the imaginary part of the polarizability of the medium exceeds the real part, appear in the transition–radiation spectrum at sufficiently high energies of the charge. It is shown that in this case the dependence of the boundary frequency and of the total intensity of the transition radiation on the Lorentz factor of the charge becomes stronger, changing from linear to quadratic.

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It is well known that both the total energy and the boundary frequency of the spectrum of the transition radiation produced by an ultrarelativistic charge at the interface between a nonabsorbing medium and vacuum depends linearly on the Lorentz factor γ of the charge.[1,2] It will be shown below that the transition-radiation phenomenon incorporates one more hitherto unnoticed effect.

We note for this purpose that the indicated linear dependence is actually based on the fact that the frequency spectrum of the radiation "cuts off" at the limiting frequency $\omega_b = \omega_0 \gamma$ (ω_0 is the plasma frequency of the medium), which depends linearly on γ. This circumstance, in turn, is due to the fact that in the formula for the distribution of the radiation intensity with respect to the frequency and the angles contains the characteristic difference

$$(\gamma^{-2} + \theta^2)^{-1} - \left(\frac{\omega_0^2}{\omega^2} + \gamma^{-2} + \theta^2\right)^{-1}, \qquad (1)$$

where θ is the emission angle. When $\omega_0^2/\omega^2 \ll \gamma^{-2}$ (or $\omega \gg \omega_b$) this difference is small and the radiation becomes negligible.

This is true, however, only so long as the imaginary part of the polarizability of the medium is small. When it becomes comparable with or even larger than the real part, the indicated difference gives way to the quantity

$$(\gamma^{-2} + \theta^2)^{-1} - \left(\frac{\omega_0^2}{\omega^2} + \gamma^{-2} + \theta^2 - i\frac{\mu c}{\omega}\right)^{-1}, \qquad (2)$$

where $\mu = \mu(\omega)$ is the linear absorption coefficient at the given frequency ω, and c is the speed of light. We see therefore that in order for the radiation to vanish it is necessary to satisfy now two conditions

$$\omega_0^2/\omega^2 \ll \gamma^{-2} \text{ and } \mu c/\omega \ll \gamma^{-2}. \qquad (3)$$

In other words, the radiation is small when

$$\omega \gg \max\{\omega_{\Gamma b}, \omega'_{\Gamma b}\}, \qquad (4)$$

where

$$\omega_b' = \mu c \gamma^2. \tag{5}$$

The absorption coefficient $\mu(\omega)$ has a minimum μ_{min} at frequencies with energy in the region of several meV for heavy elements and larger for light elements, after which it increases gradually with increasing ω and tends to a certain limiting value μ_{lim} (see, e.g., [3,4]). The values μ_{min} and μ_{lim} do not differ greatly from each other (they differ by an approximate factor of three for lead and by even less for light elements).

It is seen from the expressions for ω_b and ω_b' that if

$$\gamma >> \gamma_0 = \omega_0 / \mu_{min} c, \tag{6}$$

then $\omega_b' \gg \omega_b$ and the frequency spectrum of the transition radiation "cuts off" already at a new boundary frequency ω_b', which depends quadratically on γ. It follows therefore that for charged particles with a γ factor satisfying the condition (6), the total transition-radiation energy is proportional to γ^2. Numerical estimates yield for γ_0 values on the order of 4×10^7 for light substances such as carbon and 6×10^6 for lead.

We consider now a particular case of the passage of a charge through an interface between a medium and a boundary, with account taken of the absorptivity of the medium. The general formula for the distribution with respect to the frequency and angles[1] of the intensity of the transition radiation produced at such a boundary can be easily integrated in the hard-frequency region ($\omega \gg \omega_0$) with respect to the emission angle. As a result we obtain the following formula for the frequency spectrum:

$$\frac{dW}{d\omega} = \frac{e^2}{\pi c}\left\{\frac{|g|^2 - \gamma^{-4}}{|g - \gamma^{-2}|^2}\ \ln|g\gamma^2| + \left(\frac{2g''\gamma^{-2}}{|g-\gamma^{-2}|^2} - \frac{g'}{g''}\right)\operatorname{arc\,tg}\frac{g''}{g'} - 1\right\}, \tag{7}$$

where

$$g = g' + ig'', \quad g' = \frac{\omega_0^2}{\omega^2} + \gamma^{-2}, \quad g'' = -\frac{\mu c}{\omega}. \tag{8}$$

If the inequality (4) is not satisfied, then the quantity inside the curly brackets of (7) is generally speaking of the order of unity and decreases slowly with increasing ω. On the other hand, when the inequality (4) is satisfied, then this quantity becomes much less than unity and decreases sufficiently rapidly with increasing ω (see Fig. 1). Indeed, in this frequency region formula (7) can be expanded in powers of ω_b/ω and ω_b'/ω, the result being

$$\frac{dW}{d\omega} = \frac{e^2}{6\pi c}\left\{\left(\frac{\omega_b}{\omega}\right)^4 + \left(\frac{\omega_b'}{\omega}\right)^2\right\} << \frac{e^2}{\pi c}. \tag{9}$$

This means that the total energy of the transition radiation at $\gamma \ll \gamma_0$ is approximately equal to[1] $e^2\omega_0\gamma/3c$, and at it is of the order of $e^2\mu\gamma^2/\pi$ (see Fig. 2).

The foregoing influence of the absorption on the properties of the transition radiation takes place at large values of the γ factor of the charge. It is known, however, that at γ-factor values exceeding a certain critical γ_{cr} the influence of multiple scattering begins to assert itself (see, e.g., [5,6]). It is therefore of interest to ascertain which of these effects sets in earlier. Numerical esti-

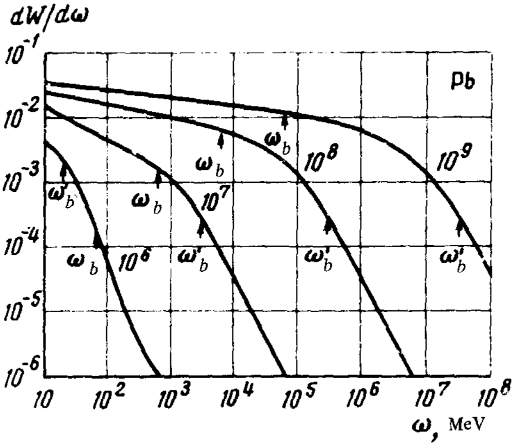

FIG. 1. Frequency spectrum of the transition radiation produced at a lead-vacuum interface for different values of the γ factor of the charge (marked on the curves). The arrows on the curves show the corresponding values of the boundary frequencies.

mates show that if the fast particle is an electron, then the influence of the multiple scattering takes place much earlier, and if the fast particle is a muon or even heavier particle, then, to the contrary, the influence of absorption sets in earlier.

For transition radiation in a plate, the picture remains qualitatively the same if the plate thickness a is of the order of or larger than the absorption length μ^{-1} up to the limiting frequencies.

In the opposite case, the interference between the radiations produced on both boundaries of the plates comes into play. The decisive role is then played by the zone in which the transition radiation is produced in the material, $z_{tr} = 2c(\omega g')^{-1}$. It is easily seen that in this case at $\omega \sim \omega_b'(\gamma > \gamma_0)$ the value of a turns

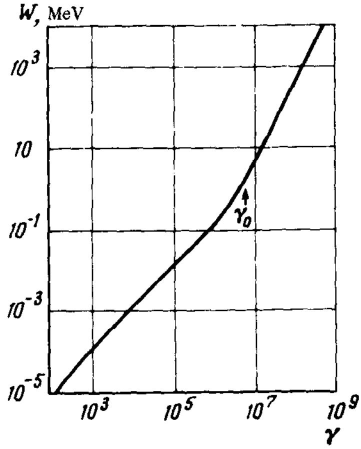

FIG. 2. Dependence of the total intensity of the transition radiation produced at a lead-vacuum boundary, starting with 1 keV, on the γ factor of the charge.

out to be smaller than z_{tr}. Therefore the spectrum at these frequencies will be suppressed, and as a result the total intensity is greatly attenuated.

We note in conclusion that the foregoing arguments are valid so long as $\hbar\omega \ll E$ (where $E = m_0 c^2 \gamma$ is the energy of the passing charged particle). Therefore when the γ factor of the charge is so large that $\omega_b' > E/\hbar$ or $\gamma > m_0 c/\hbar\mu_{tr}$, the quadratic dependence of the boundary frequency of the spectrum and of the total radiation intensity on γ should no longer hold and should again give way to a linear dependence.

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