Size-effect oscillations of the longitudinal magnetoresistance of thin cylindrical bismuth single crystals

N. B. Brandt, D. V. Gitsu, A. A. Nikolaeva, and Ya. G. Ponomarev

Moscow State University; Institute of Applied Physics of the Moldavian Academy of Sciences

(Submitted November 17, 1975; resubmitted July 30 1976)
Pis'ma Zh. Eksp. Teor. Fiz. **24**, No. 5, 304–308 (5 September 1976)

Longitudinal–magnetoresistance oscillations that are equidistant in the field and decrease in amplitude have been observed in weak magnetic fields in cylindrical bismuth single-crystal whiskers 0.2–0.8 μ thick. These oscillations are apparently caused by sample-dimension induced periodic truncation of the orbits of the electrons corresponding to quantized non-extremal sections of the Fermi surface.

PACS numbers: 72.15.Gd

We have investigated the longitudinal magnetoresistance of thin cylindrical bismuth single crystals with diameter $0.2 \le d \le 0.8\ \mu$ in magnetic fields H up to 15 kOe at helium temperatures. The single-crystal bismuth whiskers in glass insulation were obtained by casting from the liquid phase by Ulitovskiĭ's method.[1] X-ray diffraction investigations have shown that the cylinder axis of samples with diameter $d < 5\ \mu$ coincides with the ΓL direction located in the Brillouin zone in the bisector-trigonal plane and making an angle 19.5° with the bisector.

The sample diameter was obtained by calculation from the value of the resistivity of the bismuth at room temperature. This method, as shown by special electron-microscope measurements, provides a satisfactory accuracy at sample diameters larger than $0.2\ \mu$.

The ratio $\rho_{300\,\mathrm{K}}/\rho_{4.2\,\mathrm{K}}$ of the investigated samples decreased with thickness from ~ 15 at $d = 1\ \mu$ to ~ 3 at $d = 0.2\ \mu$. The plot of the longitudinal magnetoresistance $\rho(H)$ revealed at helium temperatures a negative-magnetoresistance section that shifts towards stronger magnetic fields with decreasing d.

In magnetic fields H at which the maximum electron orbit Larmor radius r_H is less than the cylinder radius $R = d/2$, normal Shubnikov oscillations, which are equidistant in the reciprocal field, are observed on the longitudinal magnetoresistance (Fig. 1). The region of the existence of the Shubnikov oscillations shifts in strong fields with decreasing d. The samples with $d > 0.4$—0.5, in fields up to 15 kOe, have two frequencies with periods $\Delta_1(1/H) = (3.8 \pm 0.4) \times 10^{-5}$ Oe^{-1} and $\Delta_2(1/H) = 8.0 \pm 1.0) \times 10^{-5}$ Oe^{-1}, which agree well with the data obtained for bulky bismuth samples with the same orientation,[2] In thinner

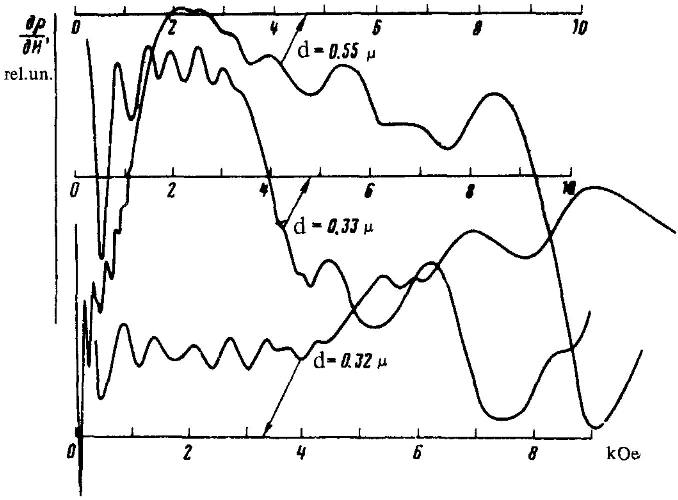

FIG. 1. Oscillation dependences of $\partial\rho/\partial H(H)$ for the case of longitudinal magnetoresistance ($T = 4.2\,^\circ$K) in cylindrical bismuth single crystals with various diameters.

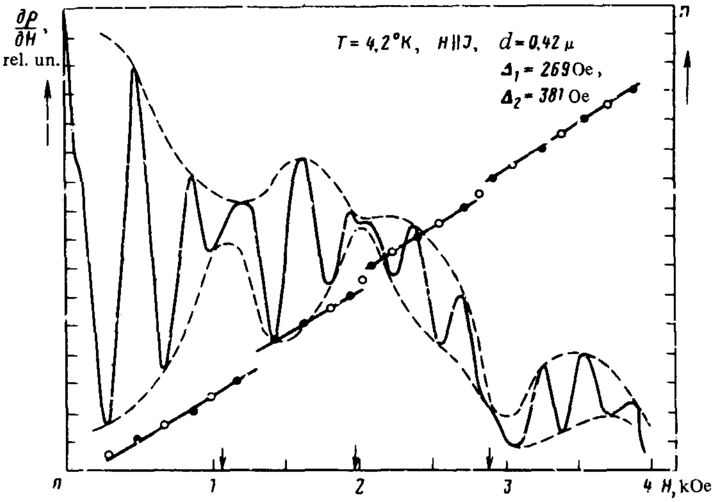

FIG. 2. Characteristic form of the size-effect oscillations of the longitudinal magnetoresistance of a sample with $d = 0.42$ at 4.2 °K, and dependence of the number of the extrema on the magnetic field. The beat nodes are marked by arrows.

samples we observed Shubnikov oscillations with only one period $\Delta_1(1/H)$. Within the limits of the experimental errors, the values of the periods $\Delta_1(1/H)$ and $\Delta_2(1/H)$ do not vary with the sample thickness.

In the initial magnetic-field intervals, all the investigated samples have revealed longitudinal magnetoresistance oscillations that attenuate with increasing field and whose parameters depend strongly on the sample diameter (Fig. 1). The oscillations have good reproducibility. The positions and shapes of the oscillation peaks do not depend on the measurement current and on the magnetic field modulation amplitude. When superheat effects arise, the amplitude of these oscillations decreases much more rapidly than the amplitude of Shubnikov oscillations in a strong field.

The characteristic features of the new oscillations consist in the following: 1) oscillations are observed in fields H weaker than the "cutoff" field $H_{\text{cut}} = cD_{\text{max}}/eD$ (D_{max} is the maximum diameter of the extremal section of the electron Fermi surface), where there are no normal Shubnikov oscillations; 2) the oscillations are superpositions of two frequencies, the period of which is constant in the direct field (one of the most clearly pronounced cases of beats of two frequencies is shown in Fig. 2); 3) the oscillation frequency increases with the sample diameter; 4) the oscillation amplitude falls off with increasing magnetic field and vanishes at the instant of the appearance of normal Shubnikov oscillations; 5) lowering the temperature from 4.2 to 2 °K causes a noticeable growth of the oscillation amplitude; the period of the oscillations is independent of the temperature.

The dependence of the frequency of the oscillations observed in the present study ($H < H_{\text{cut}}$) on the sample diameter indicate that they are due to the size effect. The clearcut picture of the beats, which was observed in most investi-

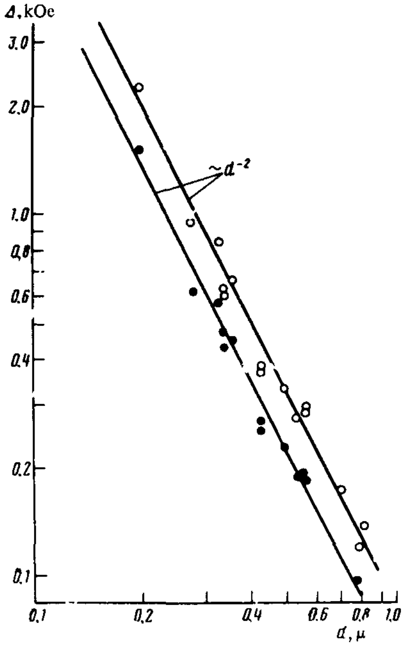

FIG. 3. Periods of the size-effect oscillations in the direct fields versus the diameter of the cylindrical bismuth single crystals.

gated samples (Fig. 2), has made it possible to determine the periods of the component frequencies with sufficient accuracy. It is seen from Fig. 3 that the periods of both frequencies of the size-effect oscillations depend on the sample diameter d like $\Delta_p \sim d^{-2}$. It follows therefore that the change $\Delta\phi = \Delta HS = \Delta_p(\pi d^2/4)$ produced in the magnetic flux through the section of the cylindrical samples when the number of the oscillations is changed by most important characteristic of the size-effect oscillations observed in the present study. For the lower $\Delta_p(d)$ branch on Fig. 3, the value of $\Delta\phi$ is $(4.3 \pm 0.4) \times 10^{-15}$ $T\,\mathrm{cm}^2$, which coincides within the limits of errors with the flux quantum value $hc/e = 4.14 \times 10^{-15}$ $T\,\mathrm{cm}^2$.

The oscillations of the magnetoresistance in weak magnetic fields at helium temperatures were observed earlier in bismuth whiskers[3,4] and in bismith films.[5] When the oscillations were registered in[3,4], hysteresis phenomena were observed, as well as a significant dependence of the position of the oscillation peaks on the measurement current. A detailed analysis of the thickness dependences of the oscillation parameters was not carried out in[3-5], so that it is impossible to compare our data with the results of[3-5].

The oscillatory effects in thin metallic cylinders in a weak magnetic field ($r_H > R$) were considered theoretically in[6,7]. It is shown in[6] that under the conditions when electrons with "grazing" orbits are reflected from the surface in a longitudinal magnetic field, magnetic surface levels are produced and give rise to oscillations of the thermodynamic quantities as functions of the magnetic flux Φ through the cylinder cross section, with a period equal to the flux quantum $\Phi_0 = hc/e$. In[7] are considered oscillatory effects in longitudinal and transverse magnetic fields for the case of diffused scattering. The oscillatory peaks in the magnetoresistance arise whenever the maximum diameter of the electron orbit corresponding to the next Landau quantum tube becomes comparable with d. In the case of longitudinal magnetoresistance and a spherical Fermi surface, the period of the oscillations as a function of the flux through the cylinder cross section is determined by the difference between the magnetic fluxes through the orbits corresponding to neighboring quantum tubes, that is, again $\Delta\phi = hc/e$. For an anisotropic Fermi surface, the period of the size-effect oscillations that are equidistant in the direct field is given by the expression[7] $\Delta_p = ch/\alpha ed^2$, where α is a parameter connecting the area of the Fermi-surface section with the square of its diameter: $S(\epsilon, p_z) = \alpha D^2(p_z)$. It follows from[7] that in the case when the Fermi surface consists of several closed surfaces with different α, the size-effect oscillations can have the character of beats. The amplitude of the oscillations predicted in[7] should decrease rapidly with increasing magnetic field.

Our present results are in qualitative agreement with the predictions of the theory of[7]. The size-effect oscillations with two frequencies are determined apparently by the section of the electron ellipsoid located in the mirror-symmetry plane containing the cylinder axis, and the two sections, of equal magnitude, of the two other electron ellipsoids, which are symmetrical about this plane.

The authors are grateful to M.M. Markus for the x-ray diffraction investigations.

[1] N.B. Brandt, D.V. Gitsu, A.M. Ioĭsher, B.P. Kotrubenko, and A.A. Nikolaeva, Prib. Tekh. Eksp. No. 3, 256 (1976).

[2] R.N. Bhargava, Phys. Rev. **156**, 785 (1967).

[3] Yu.P. Gaĭdukov and N.P. Danilova, Pis'ma Zh. Eksp. Teor. Fiz. **15**, 592 (1972) [JETP Lett. **15**, 419 (1972)].

[4] Yu.P. Gaĭdukov and N.P. Danilova, Fiz. Tverd. Tela **16**, 923 (1974) [Sov. Phys. Solid State **16**, 593 (1974)].

[5] Yu.F. Komnik, Doctoral Dissertation, Khar'kov, 1975.

[6] E.N. Borachek and G.A. Goadze, Zh. Eksp. Teor. Fiz. **63**, 1839 (1972) [Sov. Phys. JETP **36**, 973 (1973)].

[7] V.G. Peschanskiĭ and V.V. Sinolitskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **16**, 484 (1972) [JETP Lett. **16**, 344 (1972)].