Deep-inelastic scattering of leptons by hadrons and bremsstrahlung of gluons

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A valent-quark model is proposed for the description of deep-inelastic scattering of leptons by hadrons within the framework of asymptotically free theories of strong interactions. The gluons are produced only as a result of bremsstrahlung of quarks and are absent in the initial state. The model describes satisfactorily the available data and makes it possible to formulate further predictions.

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It is known that the parton model[1] describes satisfactorily the principal fundamental data on deep-inelastic scattering of leptons by hadrons. Yet there is a very serious problem which has not yet been solved in the framework of this model.

Namely, from the reduction of data on deep-inelastic scattering it follows that approximately half the nucleon momentum is carried away by gluons. On the other hand, the behavior of the form factor of the nucleon at large momentum transfers

$$F(q)^2 \sim q^{-4}$$

signifies that the number of corresponding nucleons is equal to three.[2] The large gluon contamination should violate this relation in accordance with a power law in q^2.

The spectroscopy of the low-lying baryon states also contains no indications of the existence of gluon excitations.

We propose in this article a solution of this problem and attempt to establish a connection between the valent-quark approximation and the description of deep-inelastic scattering in asymptotically free theories. The hypothesis which we shall consider reduces to the assumption that the gluons are produced in scattering of a quark by a virtual photon or W-boson, but is not contained in the initial state (a possible analog is the emission of photons when an electron is scattered in an external field).

We use for the calculations the well known technique of the moments of the structure functions of deep-inelastic scattering. Namely, within the framework of the asymptotically free field theories we can express[3] quantities of the type

$$\int_0^1 dx x^n F_2(x, Q^2) \tag{1}$$

($F_2(x, Q^2)$ is the structure function and x is the usual scaling variable) in terms of the strong-interaction coupling constant $\alpha_s = g_s^2/4\pi$, or, more accurately speaking, in terms of the ratio

$$\kappa = \alpha_s(\mu^2) / \alpha_s(Q^2) \tag{2}$$

and the matrix elements of the various operators, calculated at the normalization point

$$Q^2 = \mu^2.$$

The valent-quark approximation reduces to annihilation of the matrix elements of the operators containing fields of strange quark or gluons. In particular, when zeroth-order moments are considered, we encounter matrix elements of the energy-momentum tensors $\theta^G_{\mu\nu}$, $\theta^s_{\mu\nu}$, $\theta^q_{\mu\nu}$ of the gluon field, strange quarks, and u and d quarks, respectively. We assume that

$$\langle N|\theta^G_{\mu\nu}|N\rangle = \langle N|\theta^s_{\mu\nu}|N\rangle = 0,$$

$$\langle N|\theta^u_{\mu\nu} + \theta^d_{\mu\nu}|N\rangle = 2p_\mu p_\nu, \tag{3}$$

where p is the nucleon momentum.

Using this assumption, we can obtain for the contribution of the currents $\bar{u}\gamma_\mu u$ and $\bar{d}\gamma_\mu d$ to the structure functions:

$$\int_0^1 dx\left(F_2^{(u\to u)} + F_2^{(d\to d)}\right) = \frac{1}{3}\left(\frac{18}{25} + \frac{32}{25}\,\kappa^{-50/81} + \kappa^{-32/81}\right). \qquad (4)$$

This value corresponds, within the framework of the parton model, to the momentum carried by the partons. It is seen that the right-hand side of (4) first varies rapidly as a function of κ, and then at $\kappa \gtrsim 4$ it remains almost constant at 0.55 ± 0.05 up to unrealistically large values of κ.

A similar behavior is typical of the first moment, which characterizes the mean value of the parton momentum, and of the other quantities of interest from the point of view of comparison of theory with experiment.

Since it is known that scaling is established in deep-inelastic scattering at approximately $Q^2 \sim 2$ GeV^2, it follows that to reconcile the proposed picture with experiment we must assume

$$a_s(Q^2 = m_\pi^2) = 1. \qquad (5)$$

If we assume this value of the effective coupling constant, then the valent-quark model leads, in particular, to the following: 1) the momentum carried by the quarks is 0.55 ± 0.05 of the total nucleon momentum; 2) the momentum carried by the $(\bar{u} + \bar{d})$ antiquarks is approximately 0.03; 3) the average value of the quark momentum decreases as a result of bremsstrahlung of gluons, from 1/3 to approximately 0.22; 4) the difference between the structure functions for the scattering of the virtual photon by a proton and a neutron is equal to

$$\int_0^1 dx(F_2^{ep} - F_2^{en}) = 0.06.$$

None of these predictions contain arbitrary parameters and agree with experiment within the limits 10—20%. A remarkable property of the resultant relations is the existence of a preasymptotic region (with respect to Q^2). In this region, the theorems pertaining to the case $Q^2 \to \infty$[3] are strongly violated (for example, the equality of the cross sections of the neutrino and antineutrino interactions). However, all the quantities vary very slowly and imitate a scaling behavior. Thus, the corrolaries 1)—4) listed above pertain to the region 2 GeV^2 $\leq Q^2 \leq 300$ GeV^2, i.e., to all the values of Q^2 attainable in practice.

From among the other consequences of the models, which admit of direct comparison with experiment, we note the prediction that the antiquarks have a small average momentum ($\langle x\rangle_{\bar{q}} \sim 0.05$). This prediction can be verified for example by measuring the average values of the momentum transfer Q^2 in neutrino and antineutrino experiments:

$$1 - \frac{2\langle Q^2\rangle_{\bar{\nu}}}{\langle Q^2\rangle_{\nu}} = C\,\frac{\langle x\rangle_{\text{sea}}}{\langle x\rangle_{\text{valence}}}, \qquad (6)$$

and the contact C can also be easily calculated within the framework of the proposed model (see, e.g., [4]).

A very critical check on the model would be an independent determination of the strong-interaction coupling constant (see relation (5))

The model can also be used to calculate the cross section for the production of charmed particles in neutrino reactions. Within the framework of the standard weak-interaction scheme with four quarks, this cross section turns out to be small:

$$\frac{\sigma(\nu N \to \mu^- + \text{charm} + \text{hadrons})}{\sigma_{tot}(\nu N)} \approx \text{tg}^2\theta_c + \frac{\epsilon_s}{\epsilon_{u+d}} \approx 0.09,$$

$$\frac{\sigma(\bar{\nu} N \to \mu^+ + \text{charm} + \text{hadrons})}{\sigma_{tot}(\bar{\nu} N)} \approx \frac{3\epsilon_s}{\epsilon_{u+d}} \approx 0.15 \qquad (7)$$

and $E_{\nu,\bar{\nu}} \gg 50$ GeV. (ϵ_q is the "fraction of the nucleon momentum carried by the q quarks" and $\epsilon_q \equiv \int_0^1 dx F_2(q \to q)$. So small a contribution cannot explain the experimentally observed anomalies—it is necessary for this purpose to introduce new currents in the weak interactions.

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