Cerenkov radiation as an intense x-ray source

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It is shown that in certain substances Cerenkov radiation can be produced in the x-ray region and constitutes in this case an intense monochromatic source of radiation.

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It is known that the condition for the appearance of Cerenkov radiation at a frequency ω in a medium is that the velocity v of the charge particle exceeds the phase velocity of the light $1/\sqrt{\epsilon'(\omega)}$ ($\epsilon'(\omega)$ is the real part of the dielectric constant of the medium, $\hbar = m = c = 1$). The condition $\epsilon'(\omega) > 1$ is usually satisfied in a wide range of optical frequencies. We shall show that for certain substances this condition is satisfied also in relatively narrow intervals of x-ray frequencies near the photoabsorption edges of the internal shells of atoms.

We consider by way of example the inert gas Ar, the absorption spectrum of which was investigated in detail theoretically and experimentally.[1] The imaginary part of the dielectric constant $\epsilon''(\omega)$ is directly connected with the cross section $\sigma^{(in)}(\omega)$ for inelastic scattering of a photon by an atom and with the density N of the number of atoms by the relation $\epsilon''(\omega) = N\sigma^{(in)}(\omega)/\omega$.

Curve 1 of Fig. 1., plotted on the basis of the results of photoabsorption

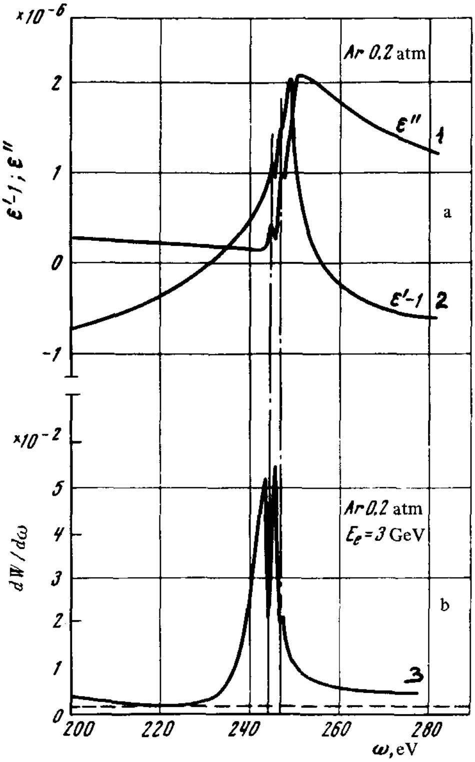

FIG. 1.

in Ar,[2] illustrates the dependence of $\epsilon''(\omega)$ of argon at normal incidence on the frequency near the frequency of the absorption L edge $\omega_L = 250$ eV. The fine structure of $\epsilon''(\omega)$ near the $L_{2,3}$ edge is due to excitation of the $L_{2,3}$ electrons on the free optical levels of the atom. Above the $L_{2,3}$ edge, the quasi-discrete absorption of the photons goes over into continuous absorption. The detailed $\epsilon''(\omega)$ dependence in the frequency region indicated in the figure makes it possible to calculate $\epsilon'(\omega)$ with the aid of the Kramers—Kronig relation. The result of the calculation is represented by the curve 2 of Fig. 1.

According to our calculations, in a rather narrow frequency interval $\Delta\omega \approx 25$ eV, where $\epsilon'(\omega) > 1$, Cerenkov radiation can be generated at frequencies $\omega \approx 250$ eV. Since the excess of $\epsilon'(\omega)$ over unity is small ($|\epsilon' - 1| \sim 10^{-6}$), only ultrarelativistic particles with energy $E \gtrsim E_c = 500$ MeV can radiate. The angles of the the Cerenkov radiation are quite small even for particles with energy greatly exceeding the threshold $E_c(\omega)$. In contrast to optical Cerenkov radiation, in the considered frequency range there are many distinguishing features in the conditions for the observation and in the properties of the emission.

Elementary estimates (see, e.g.,[3]) show that the length l_{coh} of formation of the Cerenkov photon (coherence length) is determined by the relation $l_{coh}^{-1} = \omega(\epsilon' - 1 - E^{-2})$ and is of the order of 1.0 cm under the considered conditions. This circumstance makes it possible, on the one hand, to use the microscopic characteristic $\epsilon(\omega)$ of the medium for the calculation of the radiation, even though the wavelength of the radiation may be shorter than the interatomic distance. On the other hand, at a large coherence length multiple scattering of the electrons in the medium and absorption of virtual photons exert an appreciable influence on the spectrum and on the angular distribution of the Cerenkov radiation.[4] Analysis shows that in this case the most substantial is the absorption of virtual photons over the coherence length, since the photon absorption length $l_c(\omega) = 1/\omega\epsilon''(\omega)$ can be comparable with the coherence length $l_{coh}(\omega)$ and much less than the length $l_s = 1/\sqrt{q\omega}$, which characterizes the influence of multiple scattering with mean squared angle q. As a result of absorption, Cerenkov emission can be observed in practice only behind a layer of matter with $\epsilon'(\omega) > 1$ in vacuum (or in another weakly-absorbing substance). In the interior of the medium, the Cerenkov x radiation of ultrarelativistic particles can appear in fact only in the form of an increment to the ionization losses.[4] The presence of the boundary of the medium under the condition $l_{coh}(\omega) \gtrsim l_c(\omega)$ also has a nontrivial influence on the spectrum of the Cerenkov radiation. The necessary formulas for the spectral and angular distribution $d^2W/d\omega\, d\Omega$ of the radiation emitted into the vacuum from a layer of matter with thickness $T \gg l_c$ can be obtained with the aid of the results of Garibyan[5] and take in the case of interest to us the form

$$\frac{d^2W}{d\omega\, d\Omega} = \frac{e^2}{\pi^2}\theta^2 \left| \frac{1}{\theta^2 + E^{-2} + 1 - \epsilon(\omega)} - \frac{1}{\theta^2 + E^{-2}} \right|^2, \tag{1}$$

where θ is the angle between the direction of the electron velocity and the photon momentum, $\epsilon(\omega) = \epsilon'(\omega) + i\epsilon''(\omega)$, $d\Omega$ is the solid-angle element. Elementary integration of (1) with respect to the angles lead to the result

$$\frac{dW}{d\omega} = \frac{e^2}{2\pi}\Bigg\{\left[\left(1 + \frac{2(1-\epsilon')E^{-2}}{(1-\epsilon')^2 + (\epsilon'')^2}\right)\ln\frac{(1-\epsilon'+E^{-2})^2 + (\epsilon'')^2}{E^{-4}} - 2\right]$$

$$- \frac{2}{\epsilon''}\left[1 - \epsilon' + E^{-2}\frac{(1-\epsilon')^2 - (\epsilon'')^2}{(1-\epsilon')^2 + (\epsilon'')^2}\right]\left[\frac{\pi}{2} - \operatorname{arc\,tg}\frac{1-\epsilon'+E^{-2}}{\epsilon''}\right]\Bigg\}. \tag{2}$$

In the case of weak absorption, when $\epsilon'' \ll 1 - \epsilon' + E^{-2}$, the last term in the curly brackets of (2) takes the form

$$\frac{dW^{(C)}}{d\omega} = e^2\omega\,(\epsilon' - 1 - E^{-2})\,\Theta(\epsilon' - 1 - E^{-2})\,l_c(\omega) \tag{3}$$

and constitutes the intensity of the Cerenkov radiation from the absorption length $l_c(\omega)$, calculated from the known Tamm-Frank formula.

The remainder of the total intensity is equal in this case to the transition-radiation intensity and is much smaller than the last term at not too high energies, when $\ln[1 + E^2(1-\epsilon')] \lesssim 10$.

The intensity of the emission from a layer of argon of thickness $T \gg l_c$, calculated from formula (2), is represented by curve 3 of Fig. 1. According to (1), the angle width of the Cerenkov radiation is not larger than the diffraction angle over the absorption length $\theta_d = \sqrt{\epsilon''}$. For comparison, the dashed line in the figure shows the spectral density of the synchrotron radiation from a section of the orbit, on which the electron is rotated by the magnetic field through an angle E^{-1}. The parameters of the synchrotron were chosen to be optimal for radiation in the region $\lambda \approx 50$ Å.

Thus, Cerenkov radiation can constitute an intense monochromatic x-ray source directed at angles $\theta \pm \Delta\theta (\theta = 10^{-3}, \Delta\theta \approx 10^{-4})$ and having a wavelength $\lambda \approx 50$ Å($\omega \approx 250$ eV, $\Delta\omega \approx 25$ eV).

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