Photoneutron reactions near the threshold and the optical model of the nucleus

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A quantitative interpretation of the valence mechanism of the (γn) reaction near threshold is presented. The formulas for the partial $E1$ radiative strength functions and for the nonresonant part of the cross section are expressed in terms of the shell and optical models.

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In connection with the recent progress in the experimental techniques, the (γn) and the inverse reactions are intensively investigated in the region of overlapping neutron resonances. The transition to the optical model, which can be effected by the methods of the theory of finite Fermi systems[1] makes it possible, in particular, to present a quantitative interpretation of the valence mechanism of the radiative capture of neutrons.[2] This problem breaks up into two parts: 1) parametrization of the S-matrix elements corresponding to elastic scattering of the neutrons and the (γn) reaction, S_{nn} and $S_{\gamma n}$, followed by averaging the quantities S_{nn}, $S_{\gamma n}$, $|S_{\gamma n}|^2$ over an energy interval containing many neutron resonances; 2) derivation of formulas for the average values in terms of the shell and optical models. (We confine ourselves to the case of practical importance, when the elastic neutron channel is the principal channel of the neutron-resonance decay, so that $|S_{nn}|^2=1$, and the photo-absorption cross section coincides with the cross section of the (γn) reaction.

We assume that the neutron resonances correspond to simple poles of the scattering matrix. In the energy interval near one of the neutron resonances we represent the quantities S_{nn} and $S_{\gamma n}$ in the form

$$S_{nn}(E) = e^{2i\xi}\left\{1 - \frac{i\gamma_{nc}}{E - E_c + \frac{i}{2}\gamma_{nc}}\right\}, \tag{1a}$$

$$S_{\gamma n}(E) = e^{i\psi}\left\{|S_{\gamma n}^{bg}| - e^{i\phi}\frac{i\gamma_{\gamma c}^{1/2}\gamma_{nc}^{1/2}}{E - E_c + \frac{i}{2}\gamma_{nc}}\right\}. \tag{1b}$$

Here E is the neutron energy, $\gamma_{nc}^{1/2}$ and $\gamma_{\gamma c}^{1/2}$ are the amplitudes of the neutron and radiative widths of the resonance; the quantity $S_{\gamma n}^{bg}|^2=\sigma_{\gamma n}^{bg}/g\pi\lambdabar^2$ determines the nonresonant part of the cross section of the (γn) reaction. When the neutron strength functions $S_n=\gamma_n/d$ (γ_n is the average neutron width and d is the average energy interval between resonances with definite values of spin and parity) are not small, formulas (1) must be modified in such a way as to take into account the contribution of the neighboring resonances without violating unitarity of the S matrix. Bearing in mind the subsequent averaging, such a modification can be realized by making the substitutions

$$E - E_c \to \frac{d}{\pi}\,\mathrm{tg}\frac{\pi}{d}(E - E_c);\quad \frac{1}{2}\gamma_n \to \frac{d}{\pi}\,\mathrm{th}\frac{\pi\gamma}{2d}.$$

(We have used a model of equidistant resonances.) Taking this remark into account, we obtain the average values $\overline{S}_{nn}$ and $\overline{S}_{\gamma n}$:

$$\overline{S}_{nn} = e^{2i\xi - 2\eta},\quad \eta = \tfrac{1}{2}\pi S_n, \tag{2a}$$

$$\overline{S}_{\gamma n} = e^{i\phi}\{|S_{\gamma n}^{bg}| - e^{i\phi}(1 - e^{-2\eta})(S_\gamma/S_n)^{1/2}\}. \tag{2b}$$

Formulas (2a) coincides with the result of[3]. Formula (2b) was obtained under the assumption $\overline{\gamma_{\gamma c}^{1/2}\gamma_{nc}^{1/2}}=\gamma_\gamma^{1/2}\gamma_n^{1/2}$, so that the radiative strength function for the partial transition $S_\gamma=\gamma_\gamma/d$ describes the valence part of the radiative width, which correlates with the neutron width. Under this assumption, the expression for the average cross section of the (γn) reaction takes in accordance with (1b) the form

$$\bar{\sigma}_{\gamma n}/g\pi\lambda\!\!\!^{-}{}_{\gamma}^{2} = |\overline{S_{\gamma n}}|^2 = |S_{\gamma n}^{bg}|^2 + 2(1 - e^{-2\eta})\,[S_\gamma/S_n - \cos\phi\,|S_{\gamma n}^{bg}|(S_\gamma/S_n)^{1/2}]. \quad (3)$$

It follows from (2) and (3) that the fluctuation cross sections for elastic scattering and the (γn) reaction are determined by the strength functions:

$$\sigma_{nn}^{fl}/g\pi\lambda\!\!\!^{-}{}_{n}^{2} = 1 - |\bar{S}_{nn}|^2 = 1 - e^{-4\eta}, \quad (4a)$$

$$\sigma_{\gamma n}^{fl}/g\pi\lambda\!\!\!^{-}{}_{\gamma}^{2} = |\overline{S_{\gamma n}}|^2 - |\overline{S_{\gamma n}}|^2 = (1 - e^{-4\eta})(S_\gamma/S_n). \quad (4b)$$

Using the basis of shell model, we can change over to the optical model by averaging the amplitudes of the elastic scattering of the nucleon or of the γ photon by the nucleus over the compound-nucleus states.[4,5] As applied to the elastic scattering of nucleons, the conclusion is that the average amplitude of the scattering can be calculated with the aid of the optical model with Hamiltonian $h(r) = h_0(r) + \Delta h(r)$, where h_0 is the Hamiltonian of the shell model and $\mathrm{Im}\Delta h < 0$. Let ξ and η be respectively the real and imaginary parts of the scattering phase shift of the nucleon in the optical potential. Then ξ can be identified, in accordance with (2a), with the nonresonant elastic-scattering phase shifts, and η determines the neutron strength function.

In the case of an $E1$ transition of the valence neutrons, the average amplitude of the (γn) reaction, which determines the so called optical cross section, is calculated in the following manner with the aid of the optical model[4]:

$$\sigma_{\gamma n}^{opt}/g\pi\lambda\!\!\!^{-}{}_{\gamma}^{2} = |\overline{S_{\gamma n}}|^2 = K_{ab}\,|\int \chi_E^{(+)}(r)\,r\,\chi_b(r)\,dr|^2. \quad (5)$$

Here $\chi_E^{(+)}$ is the optical-model wave function of the neutron in the continuum: $(h - E)\chi_E^{(+)} = 0$; χ_b is the shell wave function of the neutron in the bound state; $(h_0 - E_b)\chi_b = 0$; $E = E_\gamma + E_b$; K_{ab} is a kinematic factor. The formula for the valence part of the average cross section of the dipole photoabsorption was obtained in[5] via a transition to the optical model in the expression for the corresponding polarizability of the nucleus. Using this expression, and also relations (2a), (4), and (5), we obtain the following formula for the valence part of the partial $E1$ radiative force function in terms of the shell and optical models:

$$S_\gamma/S_n = K_{ab}(1 - e^{-4\eta})^{-1}\{-2\mathrm{Im}\int \chi_b(r)\,r\,G_a^{(+)}(r, r'; E)\,r'\chi_b(r')\,dr\,dr'$$

$$-2\pi|\int \chi_E^{(+)}\,r\,\chi_b\,dr|^2\}, \quad (6)$$

where $G^{(+)}(r, r', E)$ is the optical-model Green's function: $(h - E)G^{(+)} = -\delta(r - r')$.

If we use in the energy interval near the single-particle resonance the pole representations for the quantities $e^{2i\xi - 2\eta}$, $\chi_E^{(+)}$, $G^{(+)}(E)$, then we can obtain for the ratio of the strength functions (6) the expression $S_\gamma/S_n = \Gamma_\gamma/\Gamma_n$, where Γ_γ and Γ_n are the single-particle radiative and neutron widths. Although a quantitative determination of the single-particle widths contains a certain leeway, this ratio makes it possible to regard the single-particle state (quasi-discrete level) as the doorway state. According to the theory of doorway states, the quantity $S' = (\Gamma_n/\Gamma_\gamma)^{1/2} S_{\gamma n}$ constitutes, apart from a phase factor, that part of the S-matrix diagonal element which corresponds to scattering of a neutron with excitation of a discrete level, and $\gamma_{\gamma c}/\gamma_{nc} = \Gamma_\gamma/\Gamma_n$. The optical theorem $|S'(E)|^2 = -2\mathrm{Re}S'(E)$ together with formula (1b) makes it possible to obtain the missing connection between the quantities $\cos\phi$ and $|S_{\gamma n}^{bg}|$, which characterize the cross section of the (γn) reaction near the threshold:

$$2\cos\phi = (S_n/S_\gamma)^{1/2} |S_{\gamma n}^{bg}|. \tag{7}$$

Using next (2)—(7), we obtain an expression for the nonresonant cross section of the (γn) reaction:

$$\sigma_{\gamma n}^{bg}/g\pi\lambda\!\!\!^{-}{}_\gamma^2 = K_{ab} e^{2\eta} \{ 2\pi(1+\operatorname{th}\eta) \left| \int \chi_E^{(+)} r \chi_b \, dr \right|^2 + 2\operatorname{th}\eta \operatorname{Im} \int \chi_b r G_a^{(+)}(r, r', E) r' \chi_b \, dr \, dr' \}. \tag{8}$$

Formulas (2a), (6)—(8) solve the problem concerning the expression of the parameters of the partial cross sections of the photoneutron reactions near the threshold, $E1$ valence transitions, in terms of the shell and optical models. The semiquantitative analysis in[6] shows that the virtual excitation of the giant dipole resonance (the appearance of a dynamic effective charge) has little on the valence transition near the shape resonance.

We note in conclusion that in[7] the formulas for the quantities S_n, S_γ, $|S_{\gamma n}^{bg}|$ were obtained by changing over to the optical model in the expression for the average K matrix, without resorting to calculation of the fluctuation cross sections. Therefore the formulas of[7] differ from those given above. Estimates show that for nuclei in the immediate vicinity of the s resonance of the shape, the difference between the values of S_γ at energies $E \sim 10^2$ keV can reach 1 or 2 orders of magnitude.

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